G — Grazed Grains

Problem

Given $n \leq 10$ circles of radius ≤ 10 and with centers in $[0, 10] \times [0, 10]$, approximate the area of their union, up to a factor 1 ± 0.1.

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   Not too hard, but a bit of code, and there is a simpler solution: use sampling.

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3. If x of the r points are inside some circle, we estimate the area as $\frac{x}{r} \cdot 30^2$.

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4. Analysis (not needed to solve the problem): can prove that you expect a relative error around $24/\sqrt{r}$. If $r > 100$k this starts becoming small enough, and with $r = 1$ million the sampling error is very unlikely to be too large.

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Statistics at 4-hour mark: 267 submissions, 117 accepted, first after 00:07