

Problem Tutorial: “Eulerian?”

We need to check if all degrees are even.

First, let's find the total number m of edges in the first query, asking about the entire graph.

Then, do 29 iterations of the following process:

- Divide all vertices into 2 parts A and B randomly (each vertex goes to A or B with equal probability)
- Ask how many edges are in A , and how many in B , and by this deduce how many edges are there between A and B .
- If at any iteration this number is odd, report that there is no Eulerian cycle. Otherwise, there is one.

This works because the parity of the number of edges between A and B is equal to the parity of sum of degrees of vertices in A (because each edge between A and B contributes 1 to this sum, and each edge inside A contributes 2). So, if all degrees are even, the number of edges between A and B is also even. If, however, degree of some vertex X is odd, this number has probability $\frac{1}{2}$ of being even: if we move X to another group, this parity changes. Therefore, if on any of 29 runs this value is odd, there is no Eulerian cycle, else there is one (probability of failing is $\frac{1}{2^{29}}$).