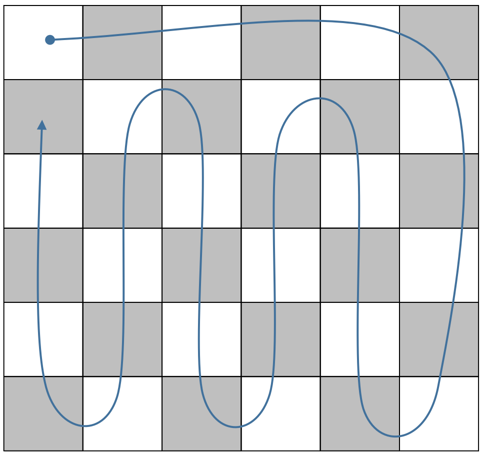
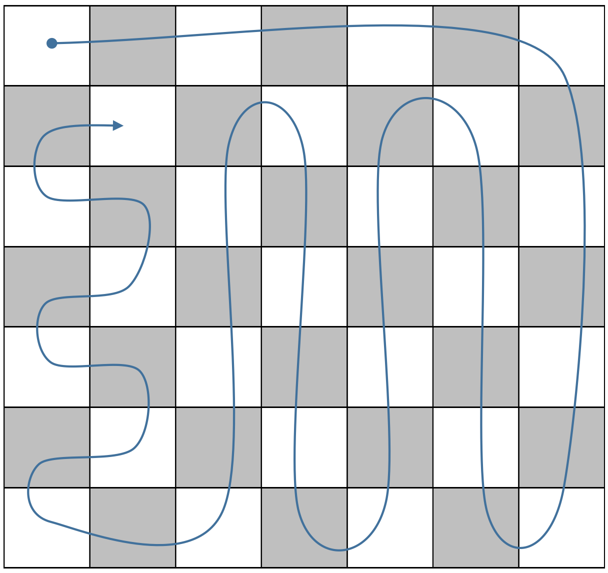
Analysis

The task concerns finding a Hamiltonian cycle in a graph, defined by some neighborhood on a rectangular network. Generally, it is an NP-complete problem and its solution needs exhausting algorithms. For small values of *N* this can be done here, too. For bigger values, some rule is to be found to realize the tour. The solution, if it exists, is even far not unique, which gives the chance for different rules to be noticed. A usual way is to split the given graph into smaller subgraphs, each with existing Hamiltonian path and after walking it, to provide a legal transition to the next subgraph. We use this method here as well.

The idea is to be able to construct a Hamiltonian path for a square with side of length 5, with given start and final positions. For to calculate quicker the first desired path using standard backtracking, we can often diminish the final demands to only the desired row or column, setting free the other dimension, as it, more often than not, does not matter. What counts is to get correctly to the next subgraph – a 5×5 square. If we can solve subtasks of this kind, we can divide the big board into smaller 5×5 squares, every one of which can be travelled without leaving its boundaries, finishing the path in a place, which assures a correct move to the next 5×5 square. If we denote the number of columns (and rows) of 5×5 squares with *T*, a Hamiltonian tour on it is easy to be seen. In the tours shown below every square is in fact a 5×5 square:



For even *T*



For odd *T*

It appears that we can use a small number of different Hamiltonian paths on a 5×5 square, less than 20. This makes it possible to remember them and reuse the found ones. They are identified by their start and end positions, as, once again, the final row or column usually do not matter.

Nothing remains but to take into consideration the necessity of leaving “steps” in the top left 5×5 corner: one or two unvisited little squares, leading to the top left little square. This should be the end of the path, transforming it into a cycle. One little square is enough for “stepping” when *T* is even, and, if *T* is odd, at least two little squares are needed. Of course, this is true for “natural” walks like the one above. As one can guess, the number of suitable paths on the board can be large.