

Problem

Given productivity values ℓ_i, f_i of n coders and productivity ℓ, f of consultant for t -hour long project, is there a weighted average of coders such that $\ell_{\text{avg}} \geq \ell/t$ and $f_{\text{avg}} \geq f/t$? Handle many queries like this, interleaved with some of the n coders leaving.

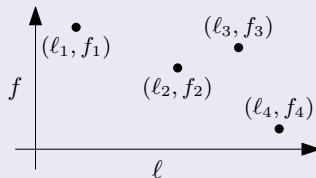
H — Hiring Help

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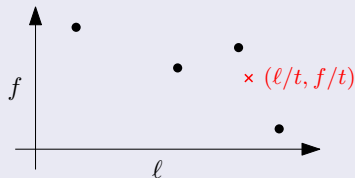
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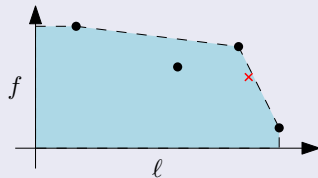
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- 3 The weighted average exists if and only if q is below the upper side of the convex hull of P .



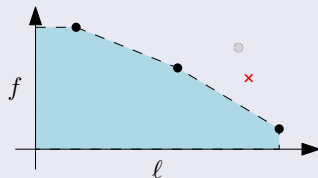
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- 4 Coders leaving corresponds to points being removed, leading to the convex hull changing.



Reformulated Problem

Given set of points (x, y) , maintain upper side of its convex hull, under removals of points and queries about whether other points (x^*, y^*) are below the hull.

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- 1 The hull is a piecewise linear function, represent it as a sorted set of points $(x_1, y_1), (x_2, y_2), \dots, (x_t, y_t)$ where $x_1 < x_2 < \dots < x_t$ and $y_1 > y_2 > \dots > y_t$.

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- 2 For a query (x^*, y^*) , find index i such that $x_{i-1} < x^* \leq x_i$ and check if (x^*, y^*) is below line from (x_{i-1}, y_{i-1}) to (x_i, y_i) .

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Given set of points (x, y) , maintain upper side of its convex hull, under *additions* of points and queries about whether other points (x^*, y^*) are below the hull.

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- 3 Handling removals can be done, but if we instead run the events in reverse order, the removals become *additions*, which are easier to handle.

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Handling additions

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Statistics at 4-hour mark: 24 submissions, 7 accepted, first after 01:42