

# E: Endgame

Problem Author: Nils Gustafsson



## Problem

Given the location of a piece on an  $n \times n$  playing board and  $n$  types of moves ( $n \leq 10^5$ ). Find a position on the board that the piece cannot reach within two moves.

## Solution

- Simpler question: Given a specific position, can the piece reach that position within two moves?
  - BFS/DFS will take  $O(n^2)$  time, which is too slow.
  - Bidirectional search:
    - $F$ : the set of positions that the piece can reach within one move.
    - $B$ : the set of positions that can reach the target position within one move.
    - $F$  and  $B$  intersect iff. the piece can reach the position within two moves.
    - These sets can be constructed and intersected in  $O(n \log n)$  time.
- Asking this question for all  $n^2$  positions on the board is way too slow.
  - Do we have to try all of them?

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### Solution

- In the worst case, the piece can reach at most approx.  $n^2/2$  positions on the board within two moves.
- If we pick a random position on the board, the piece can reach that position within two moves with probability at most  $1/2$ .
  - Repeating this  $k$  times, the probability that the piece can reach all of them within two moves is at most  $1/2^k$ , which quickly tends to 0.
- Run bidirectional search on 30 random positions.

### Gotchas

- The piece is not allowed to move off the playing board.
- When  $n \in \{2, 3\}$ , the piece may be able to reach all the positions within two moves.

Statistics: 210 submissions, 37 + ? accepted