Computation Master

Input file: `standard input`
Output file: `standard output`
Time limit: 1 second
Memory limit: 1024 megabytes

Little Cyan Fish was learning an algorithmic lecture at the National Olympiad in Fishing Winter Camp (WC). In the lecture, a mysterious lecturer talked about matrix multiplication, which is a binary operation that produces a matrix from two matrices.

For matrix multiplication, the number of columns in the first matrix must be equal to the number of rows in the second matrix. The resulting matrix, known as the matrix product, has the number of rows of the first and the number of columns of the second matrix. The product of matrices A and B is denoted as $C = AB$.

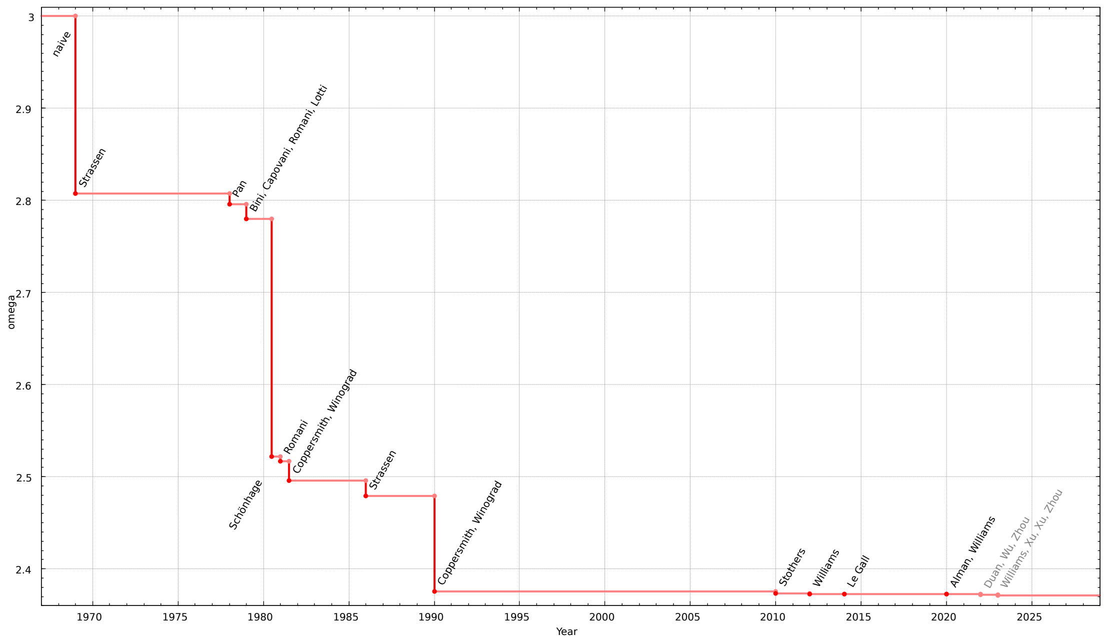
$$C_{i,j} = \sum_k A_{i,k} B_{k,j}$$

The mysterious lecturer especially noted the computational complexity of matrix multiplication, which dictates how quickly the operation of matrix multiplication can be performed. Directly applying the mathematical definition of matrix multiplication gives an algorithm that requires $O(n^3)$ field operations to multiply two $n \times n$ matrices over that field. The first faster algorithm to be discovered was Strassen's algorithm, devised by Volker Strassen in 1969 and often referred to as "fast matrix multiplication".

The "matrix multiplication exponent", usually denoted ω, is the smallest real number for which any two $n \times n$ matrices over a field can be multiplied together using $n^{\omega+o(1)}$ field operations. This notation is commonly used in algorithms research, so that algorithms using matrix multiplication as a subroutine have bounds on running time that can update as bounds on ω improve. Using a naive lower bound and schoolbook matrix multiplication for the upper bound, one can straightforwardly conclude that $2 \le \omega \le 3$. Whether $\omega = 2$ is a major open question in theoretical computer science, and there is a line of research developing matrix multiplication algorithms to get improved bounds on ω.

In the Winter Camp, the mysterious lecturer showed the following table to illustrate the timeline of matrix multiplication exponent. The best announced bound on the asymptotic complexity of a matrix multiplication algorithm was given by Williams, Xu, Xu, and Zhou in July 2023, announced in a preprint.

Year	**Bound on ω**	**Author(s)**
1969	2.8074	Strassen
1978	2.796	Pan
1979	2.780	Bini, Capovani, Romani
1981	2.522	Schönhage
1981	2.517	Romani
1981	2.496	Coppersmith, Winograd
1986	2.479	Strassen
1990	2.3755	Coppersmith, Winograd
2010	?	Stothers
2012	?	Williams
2014	?	Le Gall
2020	?	Alman, Williams
2022	?	Duan, Wu, Zhou
2023	?	Williams, Xu, Xu, and Zhou



However, Little Cyan Fish didn't listen carefully to the mysterious lecturer's lesson. After the Winter Camp, he forgot the information in the last few lines of this table. He wonders that, as of Jan 12th 2024, what is the best announced upper bound of ω?

Input

There is no input in this problem.

Output

Output a single line contains a single real number, indicating the best announced bound of ω as of Jan 12th 2024.

Your answer will be considered correct if its absolute error does not exceed 10^{-3}. Formally speaking, suppose that your output is a and the jury's answer is b, your output is accepted if and only if $|a-b| \le 10^{-3}$.

Example

standard input	standard output
<no input>	2.1145141919810

Note

Please note that the sample output is NOT the correct answer! It only serves the purpose of showing you the output format.