ICPCCamp 2016

Day 8

Makoto Soejima's Contest #4

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Problem A. 2016

Happy New Year! The integer 2016 has exceptionally many divisors.

Let $d(n)$ be the number of divisors of n. For example, $d(12) = 6$ because it has 6 divisors: 1, 2, 3, 4, 6, and 12. A positive integer x is called *divisorful*, if the number of positive integers y that satisfy both $y < x$ and $d(y) > d(x)$ is at most one. For example, 2016 is a divisorful number because among integers smaller than 2016, only 1680 has more divisors than 2016.

You are given an integer K. Compute the K-th (1-based) smallest divisorful number. If such number is strictly greater than 10^{18}, print -1 instead.

Constraints

- $1 \leq K \leq 10^9$

Input

K

Output

Print the answer in a single line.

Sample Input 1

```
10
```

Sample Output 1

```
14
```

The smallest divisorful numbers are 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, ...

Sample Input 2

```
1000000000
```

Sample Output 2

```
-1
```

Problem B. Airports

Snuke is the owner of N airports. The coordinates of the i-th airport are (x_i, y_i). Snuke chooses a constant D, and for each pair of two airports p and q, add a flight between these two airports if the Manhattan distance between p and q is at least D. Compute the maximal D that makes the airports connected (that is, any airport is reachable from any other airports by using one or more flights).

Note that the Manhattan distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is defined as $|x_1 - x_2| + |y_1 - y_2|$.

Constraints

- $2 \leq N \leq 100000$
- $0 \leq x_i, y_i \leq 10^9$
- No two airports are at the same position.
- All numbers in the input are integers.

Input

N
$x_1 \quad y_1$
$\vdots$
$x_N \quad y_N$

Output

Print the answer in a single line.

Sample Input 1

```
6
1 7
8 5
6 3
10 3
5 2
6 10
```

Sample Output 1

```
9
```

Problem C. Jump

Snuke is standing on an infinitely long road. The position on this road is represented by a real number.

Snuke can perform N types of jumps. The jump of type i is symmetric with respect to the point a_i. (i.e., if he performs this jump at the point x, he will jump to $2a_i - x$).

You are given Q queries. In the i-th query, you are asked to compute the minimum number of jumps Snuke must perform to go from s_i to t_i. If t_i is unreachable from s_i by performing a series of jumps, print -1 instead.

Constraints

- $1 \le N \le 200$
- $0 \le a_1 < ... < a_N \le 10000$
- $1 \le Q \le 100000$
- $0 \le s_i, t_i \le 10000$
- All numbers in the input are integers.

Input

N
a_1
$\vdots$
a_N
Q
$s_1 \quad t_1$
$\vdots$
$s_Q \quad t_Q$

Output

For each query, print the answer in a single line.

Sample Input 1

```
4
1
2
4
7
10
2 3
5 6
6 0
3 7
```

```
10 3
7 6
5 5
2 10
4 10
10 10
```

Sample Output 1

```
-1
-1
2
2
-1
-1
0
3
1
0
```

Problem D. Merge

Snuke wants to create an array R by merging two arrays P and Q. Formally, the array R is obtained in the following way:

- Initially, the array R is empty.
- While at least one of P and Q is non-empty, choose a non-empty array (P or Q), pop its leftmost element, and attach it to the right end of R.

You are given P and Q. Compute the number of possible distinct R Snuke can create, and print the answer modulo 1,000,000,007. Note that both P and Q are permutations of $1, \ldots, N$.

Constraints

- $1 \leq N \leq 2000$
- Each of P and Q is a permutation of $1, \ldots, N$.

Input

N
$P_1 \ \cdots \ P_N$
$Q_1 \ \cdots \ Q_N$

Output

Print the answer in a single line.

Sample Input 1

```
4
3 1 2 4
3 1 2 4
```

Sample Output 1

```
14
```

Sample Input 2

```
10
5 7 3 1 6 4 2 10 9 8
2 8 9 1 5 6 10 4 3 7
```

Sample Output 2

```
127224
```

Problem E. Mirror Rice Cake

Mirror Rice Cake (a stack of rice cakes) is a famous Japanese food that is used for celebrating a new year.

Snuke has N rice cakes to create a Mirror Rice Cake. The weight of the i-th rice cake is a_i. He wants to create a Mirror Rice Cake by choosing some of these rice cakes and stacking them in some order. Additionally, it must satisfy the following constraint: for each rice cake in the stack, the total weight of all rice cakes above it must be strictly smaller than its own weight.

Compute the maximum possible number of rice cakes he can use to create a Mirror Rice Cake.

Constraints

- $1 \leq N \leq 1000$
- $1 \leq a_i \leq 10^9$
- All numbers in the input are integers.

Input

N
a_1
$\vdots$
a_N

Output

Print the answer in a single line.

Sample Input 1

```
5
3
20
5
8
6
```

Sample Output 1

```
3
```

For example, stack three rice cakes of sizes $3, 5, 20$ from top to bottom.

Problem F. Number Cards

Snuke has N number cards. On the i-th card, a positive integer a_i is written, and the color of this card is c_i (in this problem we represent colors by integers).

Snuke has the following hypothesis about the coloring scheme of these cards:

- $1, \ldots, M$ are colored by the same color.
- $M+1, \ldots, 2M$ are colored by the same color, and this color is different from the color used for $1, \ldots, M$.
- $2M+1, \ldots, 3M$ are colored by the same color, and this color is different from the colors used for $1, \ldots, 2M$.
- $3M+1, \ldots, 4M$ are colored by the same color, and this color is different from the colors used for $1, \ldots, 3M$.
- and so on.

How many positive integers M are consistent with all the cards he has? If the number of possibilities of M is infinite, print -1.

Constraints

- $1 \leq N \leq 20$
- $1 \leq a_1 < ... < a_N \leq 10^9$
- $1 \leq c_i \leq 20$

Input

N
$a_1 \ \ c_1$
$\vdots$
$a_N \ \ c_N$

Output

Print the answer in a single line.

Sample Input 1

```
4
27 2
2000 4
2015 4
2100 1
```

Sample Output 1

```
277
```

Sample Input 2

```
3
1 1
2 2
3 1
```

Sample Output 2

```
0
```

Problem G. Paint

Snuke wants to paint a picture. His picture is simply a sequence of black and white cells.

Initially, he prepares a strip of white paper, and divides it into N cells. Then, he performs K operations. In the i-th operation, he chooses consecutive a_i cells, and paint these cells black. All chosen white cells will be black, and all chosen black cells will remain unchanged.

How many distinct pictures can he draw? Compute the answer modulo 1,000,000,007. Two pictures are considered different if the color of at least one cell is different. We don't rotate pictures - for example, (black - black - white) and (white - black - black) are different pictures.

Constraints

- $1 \le N \le 10^9$
- $1 \le K \le 4$
- $1 \le a_i \le N$

Input

$N \quad K$
a_1
$\vdots$
a_K

Output

Print the answer in a single line.

Sample Input 1

```
10 2
1
1
```

Sample Output 1

```
55
```

You can draw all pictures that have either one or two black cells.

Sample Input 2

```
1000000000 4
2015
2015
123456789
27
```

Sample Output 2

```
782767239
```

Problem H. Random Walk

There is an infinitely large 2-dimensional square grid. The coordinates on this grid are represented by a pair of integers (i, j).

Snuke wants to do a random walk. He starts from $(0, 0)$ and makes N steps. When he is at (i, j), his position after the next step will be one of $(i-1, j), (i, j-1), (i, j+1), (i+1, j)$. Each of these possibilities will happen with probability $1/4$.

Let E be the expected number of visited cells during the random walk. Compute the value $E \times 4^N$ modulo M (this value is guaranteed to be an integer). Note that $(0, 0)$ is always considered visited.

Constraints

- $1 \leq N \leq 5000$
- $10^9 \leq M \leq 2 \times 10^9$

Input

$N \quad M$

Output

Print the answer in a single line.

Sample Input 1

```
2 1000000007
```

Sample Output 1

```
44
```

Sample Input 2

```
2015 2000000000
```

Sample Output 2

```
1892319232
```

Problem I. Robots

Snuke has N robots. They are numbered 1 through N. Initially, the robot i is placed at (x_i, y_i), and it faces the direction d_i. d_i is one of 'U', 'D', 'L', and 'R': they represent the y-plus direction, the y-minus direction, the x-minus direction, and the x-plus direction, respectively.

Initially no robots are moving. However, each robot will start moving to the direction it faces at the unit speed if it touches something (Snuke or another robot). These robots are made of strange material and they can pass through other robots. Once a robot starts moving, it keeps moving no matter what happens; even if it touches another robot, it won't change its direction and speed.

Snuke touched the robot 1 at time 0. Compute the coordinates of each robot at time T.

Constraints

- $1 \leq N \leq 100000$
- $0 \leq T \leq 10^{18}$
- $0 \leq x_i, y_i \leq 10^9$
- d_i is one of the following characters: 'U', 'D', 'L', 'R'.
- At time 0, no two robots are at the same position.
- All numbers in the input are integers.

Input

N T
x_1 y_1 d_1
$\vdots$
x_N y_N d_N

Output

Print N lines. In the i-th line, print the coordinates of the robot i at time T.

Sample Input 1

```
5 10
1 0 U
3 1 U
1 2 R
1 1 L
0 1 R
```

Sample Output 1

```
1 10
3 6
9 2
-8 1
8 1
```

Problem J. Ropes

N people are sleeping. They are numbered 1 through N. Snuke wants to connect them using $N-1$ ropes!

- The two ends of each rope must be attached to two distinct people. These two people will be directly connected by a rope.
- All people must be connected by ropes directly or indirectly.
- Exactly a_i ropes must be attached to the people i.

Compute the number of ways to connect people while satisfying all conditions above, modulo 1,000,000,007. Assume that ropes are indistinguishable (of course, people are distinguishable).

Constraints

- $2 \le N \le 100000$
- $1 \le a_i \le 3$

Input

N
a_1
$\vdots$
a_N

Output

Print the answer in a single line.

Sample Input 1

```
9
1
3
2
1
3
1
2
1
2
```

Sample Output 1

```
1260
```

Problem K. Stains

There are N stains on Snuke's desk. The coordinates of the i-th stain are (x_i, y_i).

Snuke wants to add zero or more stains and create an interesting pattern. A set of stains is interesting if the number of stains is K^2 for some integer K and they form a square grid of dimensions $K \times K$. Note that this square grid is not necessarily parallel to coordinate axis.

Formally, a square grid of dimensions $K \times K$ is the set $(a + ci + dj, b + di - cj), 0 \le i, j \le K - 1$ for some constants a, b, c, d.

Compute the minimum number of stains Snuke must add to create a square grid. Assume that the desk is sufficiently large and he can add new stains at any coordinates. All input coordinates are integers, but the coordinates of new stains don't necessarily have to be integers.

Constraints

- $1 \le N \le 100000$
- $0 \le x_i, y_i \le 10^9$
- No two stains are at the same position.
- All numbers in the input are integers.

Input

N
$x_1 \;\; y_1$
$\vdots$
$x_N \;\; y_N$

Output

Print the answer in a single line.

Sample Input 1

```
3
1 5
3 6
4 9
```

Sample Output 1

```
6
```

For example, you can add stains at the following six points: $(5, 7), (0, 7), (2, 8), (-1, 9), (1, 10), (3, 11)$.

Problem L. String Modification

Snuke received a string s as a new year present. Determine if he can convert it to his favorite string, t, by repeating the following operations zero or more times.

Operation: Choose a character from s, and insert another character right after the chosen character. The inserted character must be different from the chosen character.

For example, he can convert "abca" to "adbca" in a single operation by choosing the first 'a' and inserting a 'd' right after it. However, he can't convert "abca" to "aabca" in a similar way.

Constraints

- $1 \leq |s| \leq |t| \leq 5000$
- s and t consist of lowercase letters.

Input

s
t

Output

Print "Yes" or "No".

Sample Input 1

```
snuke
snukent
```

Sample Output 1

```
Yes
```

Sample Input 2

```
snuke
ssnuke
```

Sample Output 2

```
No
```

Problem M. Team Competition

N people wants to practice for an upcoming team competition. Snuke wants to schedule the practice, and it should satisfy the following conditions:

- The number of dates of the practice is between 1 and N^2, inclusive.
- Each day, exactly 3 of N people will participate in the practice.
- Let $f(p,q)$ be the number of dates in which both p and q will practice. The value $f(p,q)$ must be the same for all pairs of two distinct people (p,q).

Constraints

- $3 \leq N \leq 1000$

Input

N

Output

If no schedule that satisfy the conditions exists, print -1 in a single line. Otherwise, print a schedule that satisfy the conditions in the following format. (If there are multiple such schedules, you can print any.) Here, K is the number of days, and x_i, y_i, z_i are the indices of the three people who practice in day i. People are numbered 1 through N.

$$
\begin{array}{l}
K \\
x_1 \ \ y_1 \ \ z_1 \\
\vdots \\
x_K \ \ y_K \ \ z_K
\end{array}
$$

Sample Input 1

```
5
```

Sample Output 1

```
10
1 2 3
1 2 4
1 2 5
1 3 4
1 3 5
1 4 5
2 3 4
2 3 5
```

```
2 4 5
3 4 5
```