

# NWERC 2014

## Presentation of solutions

The Jury

2014-11-30

## NWERC 2014 Jury

- Per Austrin (KTH Royal Institute of Technology)
- Thomas Beuman (Leiden University)
- Jeroen Bransen (Utrecht University)
- Tommy Färnqvist (LiU)
- Jan Kuipers (AppTornado)
- Lukáš Poláček (KTH and Spotify)
- Alexander Rass (FAU)
- Fredrik Svensson (Autoliv Electronics)
- Tobias Werth (FAU)

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Given two lists of strings, match them up so that as many as possible are identical.

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Statistics: 135 submissions, 92 accepted

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## Problem

Given a tree, assign a number (the frequency) to each edge, such that every node is adjacent to at most 2 different numbers (its NIC's frequencies), and as many numbers as possible are used.

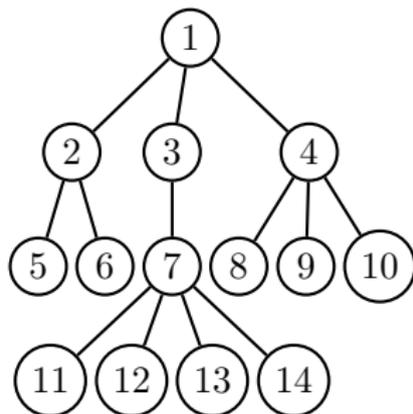
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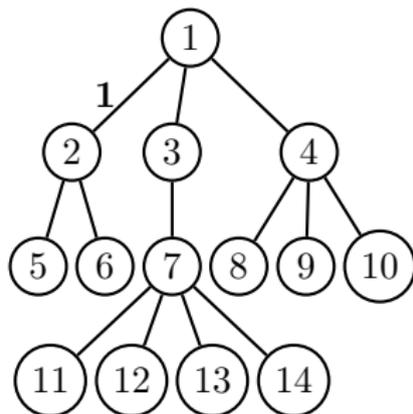
## Solution

Greedy: start somewhere, assign a new number to an edge if possible, otherwise reuse one, and then recurse.

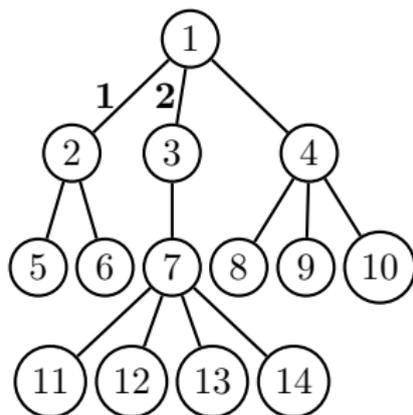
## H – Hyacinth (sample 2)



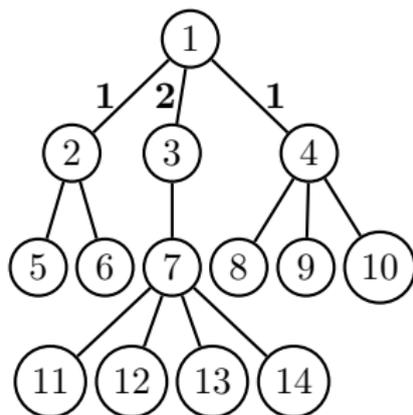
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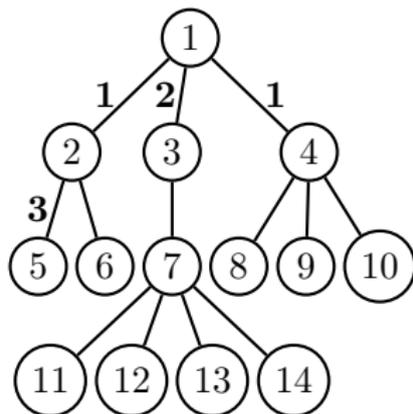
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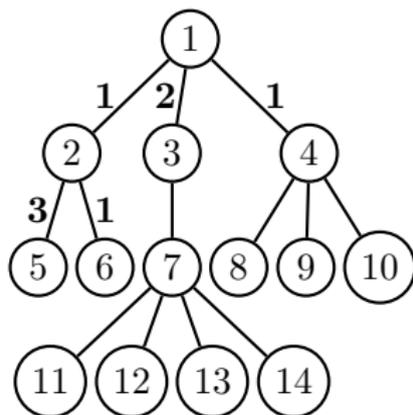
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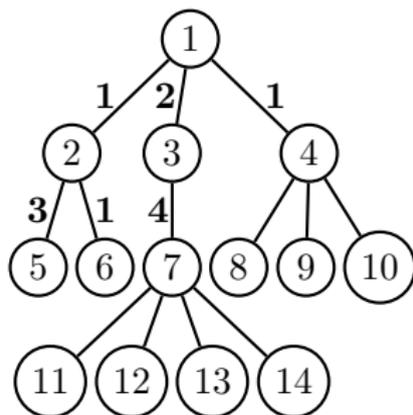
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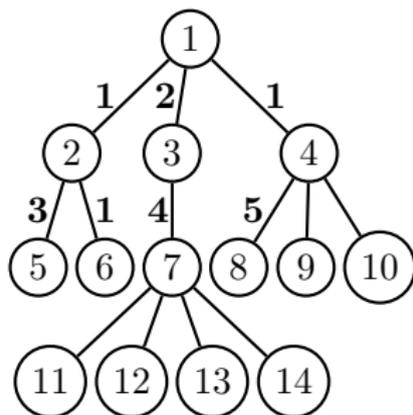
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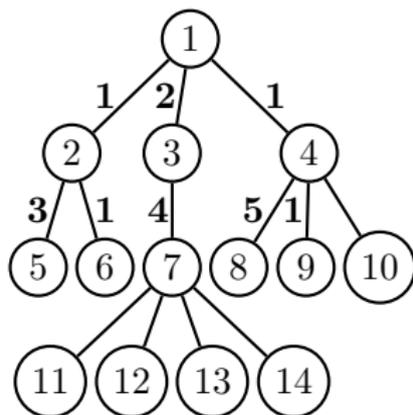
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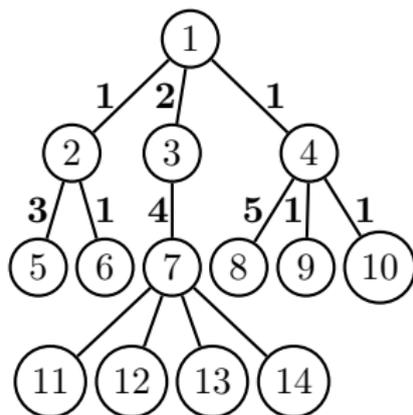
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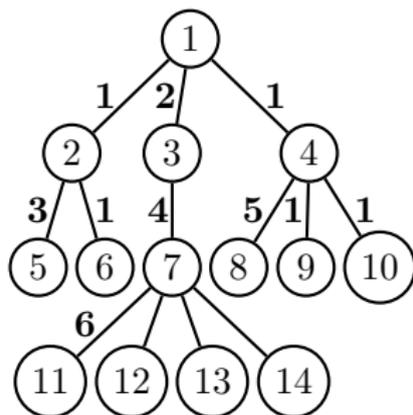
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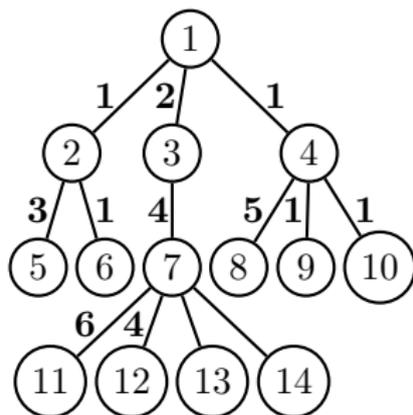
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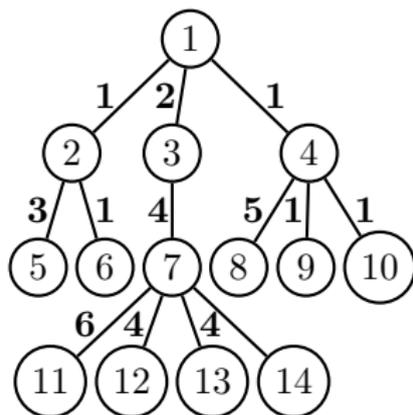
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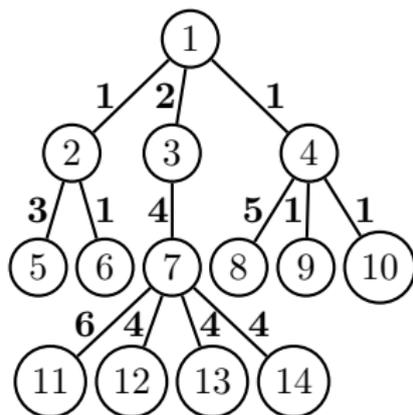
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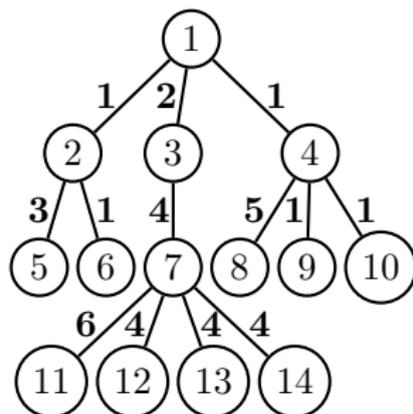
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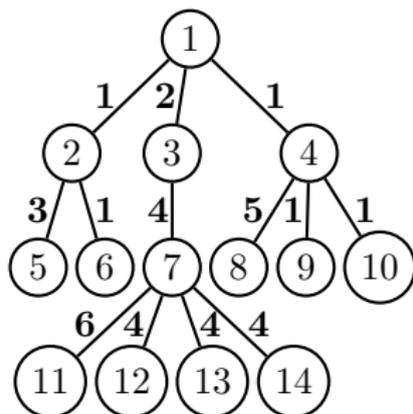


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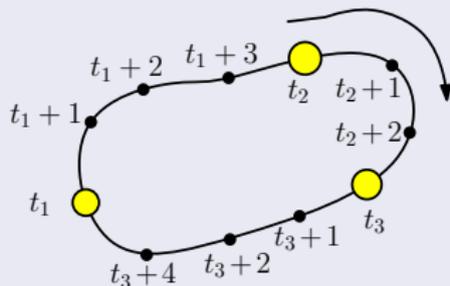
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- Time from others grow arithmetically

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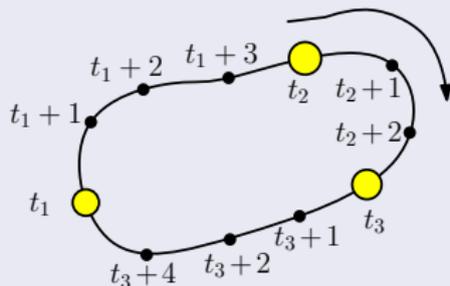


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## G – Gathering

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## Solution

- The “ideal” position is  $x^* = \text{median}(x_1, \dots, x_n)$  and  $y^* = \text{median}(y_1, \dots, y_n)$
- But might be more than distance  $d$  away from some point...

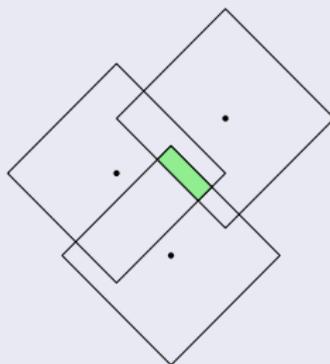
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Feasible region is intersection of diamonds, forms a “skewed diamond”



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- Key observation: there is an optimal point which either:
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- Given a candidate value for  $x^*$ , we get a range of allowed  $y$  values  $y_{lo} \dots y_{hi}$ . Best choice is median  $y$  (if in range) or  $y_{lo}$  or  $y_{hi}$ .

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- Key observation: there is an optimal point which either:
  - 1 Is a corner of the feasible region, or
  - 2 Shares  $x$ -value with an input point, or
  - 3 Shares  $y$ -value with an input point.
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Statistics: 20 submissions, ? accepted

## B – Biking Duck

### Problem

Find shortest path, possibly via some of the given bike stations or the (infinitely many possible) stations the boundary.

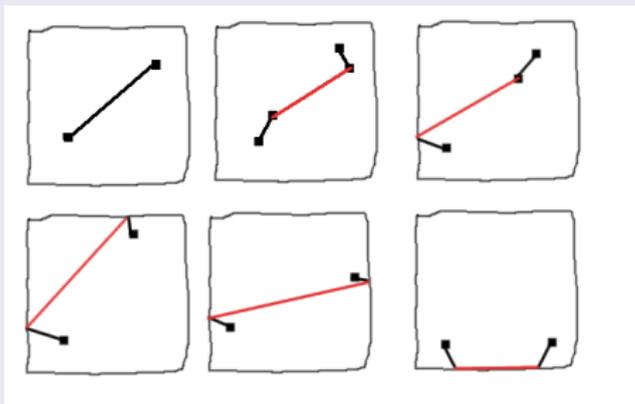
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Possible solutions use 0 or 2 stations:



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Statistics: 7 submissions, ? accepted

# A – Around the Track

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Find shortest tour around a track defined by two polygons.

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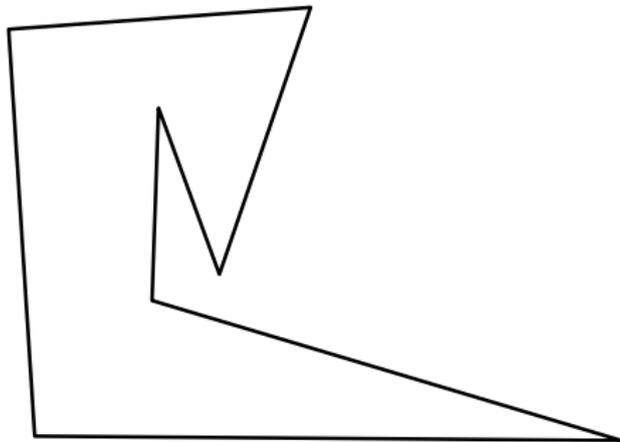
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Can also use Dijkstra, need to be careful to make a full lap around the inner polygon.

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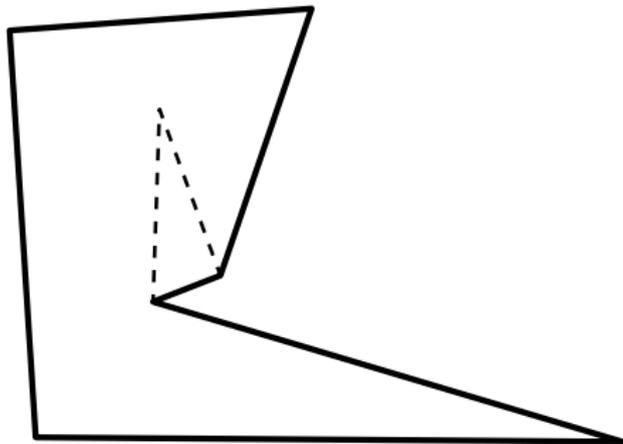
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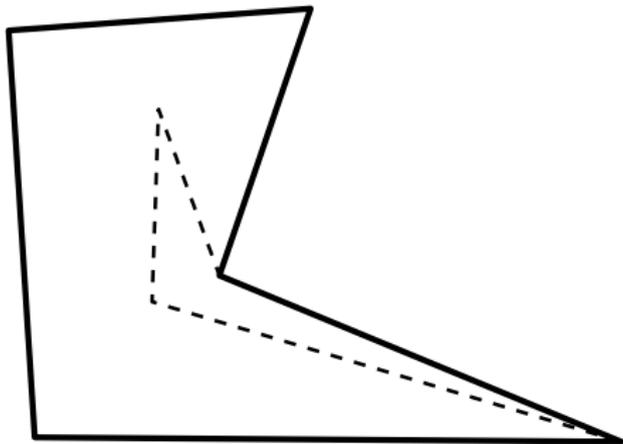
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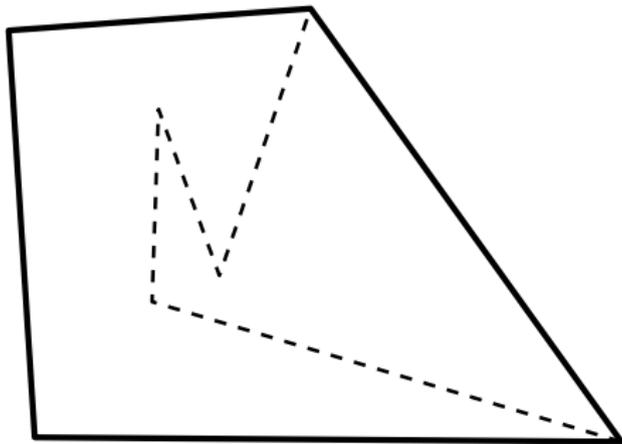
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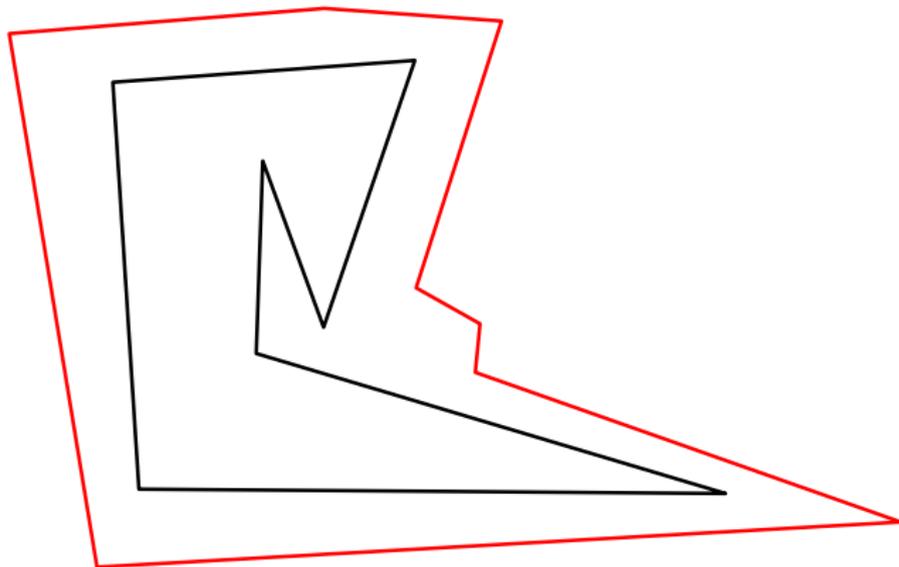
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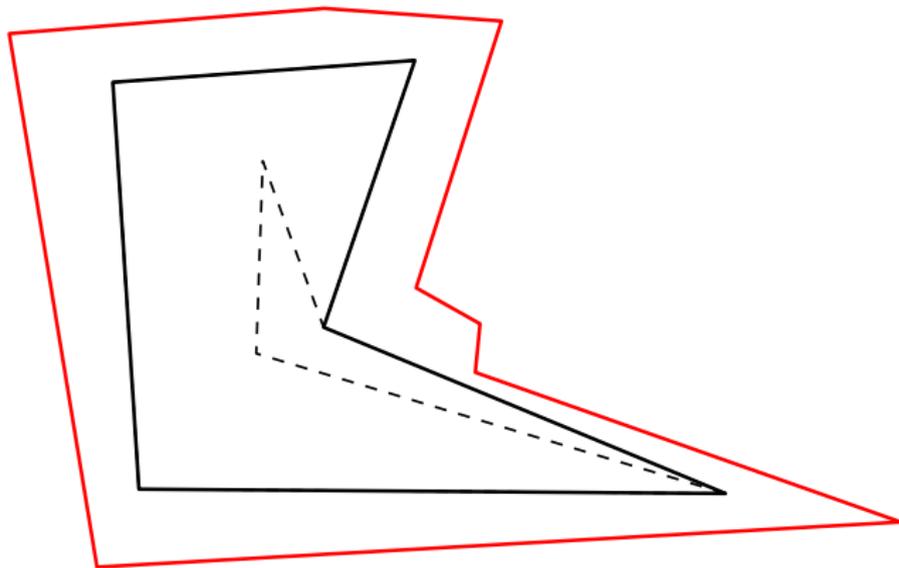
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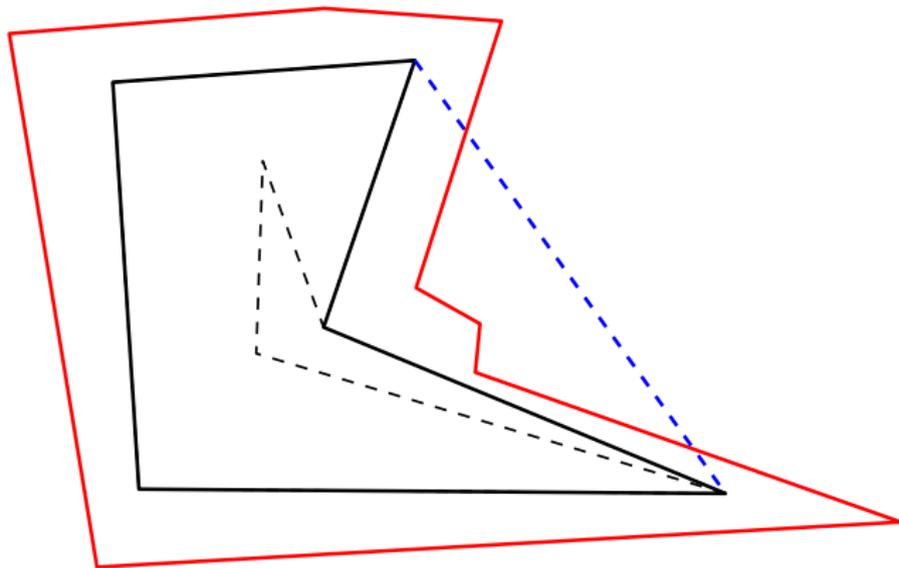
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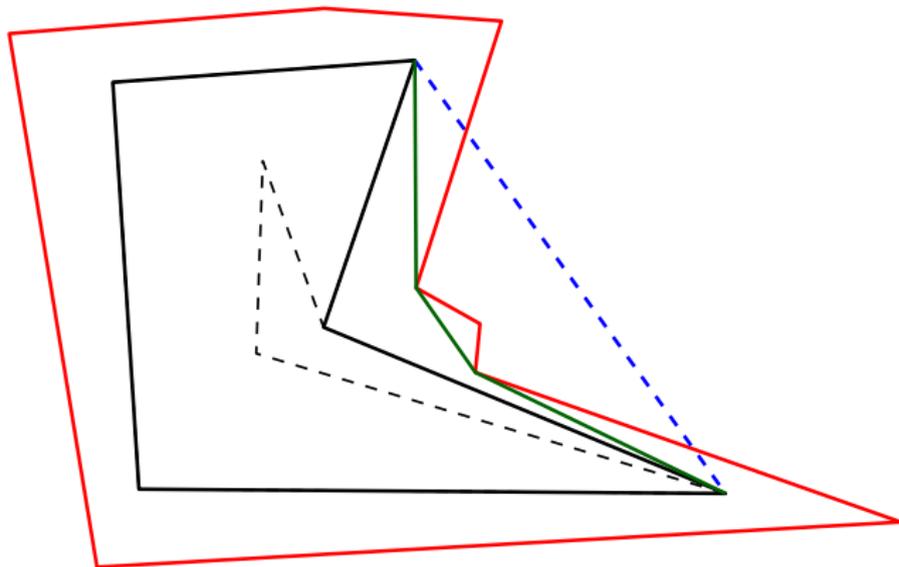
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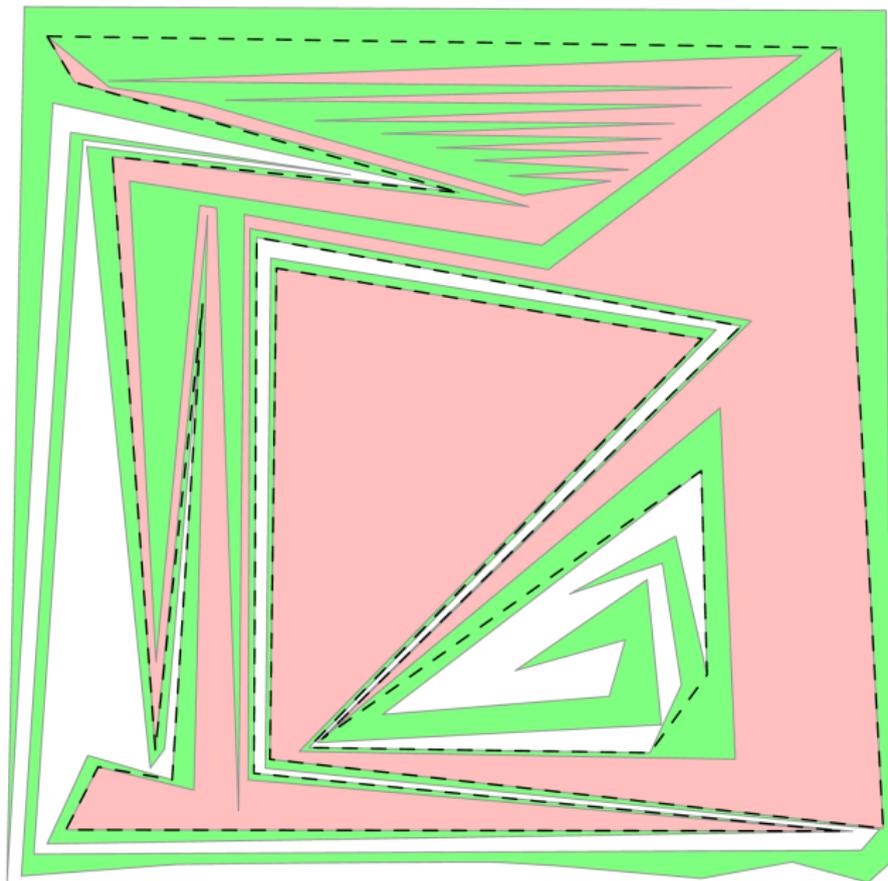
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## Random numbers produced by the jury

- 1087 number of posts made in the jury's forum.
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- 380 number of lines of code used in total by the shortest judge solutions to solve the entire problem set.
- 16.7 average number of jury solutions per problem (including incorrect ones)
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