

# 矩阵变换

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# Problem description

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0 1 0 2 3 0 0 4 0 5
2 0 3 0 0 1 0 5 4 0
4 2 1 0 0 0 3 0 5 0
0 3 0 4 0 5 0 1 2 0
1 0 0 3 2 4 5 0 0 0
```

N rows, M columns. ( $M > N$ )

Each row contains exactly one  $x$  for any  $1 \leq x \leq N$ .

Each column contains no more than one  $x$  for any  $1 \leq x \leq N$ .

# Problem description

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0 1 0 2 3 0 0 4 4 4

2 0 3 0 0 1 0 5 5 5

4 2 1 0 0 0 3 3 3 3

0 3 0 4 0 5 0 1 1 1

1 0 0 3 2 2 2 2 2 2

Truncate each row as shown above.

We want to keep the property that each column contains no more than one x.

Give an  $O(NM)$  algorithm.

# Partial credit solution

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Brute force.

# Full credit solution

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Gale-Shapley Stable Matching (or equivalent forms).

# What's Gale-Shapley Stable Matching?

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给出 $n$ 个男孩,  $n$ 个女孩.

每个男孩对所有女孩有一个序关系, 表示他对女孩的喜好程度.

每个女孩也对所有男孩有一个序关系, 表示她对男孩的喜好程度.

定义一个稳定匹配为满足下列性质的完备匹配:

- 不存在一个男孩和一个女孩, 他们没有配对, 但互相喜欢对方胜过自己的配偶.

# What's Gale-Shapley Stable Matching?

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稳定匹配一定存在!

GS Stable Matching Algorithm runs as follows:

- 每轮任选一个还未匹配的男孩, 令其向还未求婚过的他最喜欢的女孩求婚.
- 如果那个女孩还没有配偶, 或者喜欢当前男孩胜过自己的配偶, 那么她就会同意此次求婚. 否则会拒绝.
- 直至所有男孩都有配偶.

GS稳定婚姻系统确保找到一个稳定匹配.

# What about this problem?

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行是男孩, 数是女孩.

一行喜欢在这行靠前的数.

数喜欢她出现的靠后的行.