Problem Tutorial: "Hit"

Let's start with placing points so that each segment contains at least one point greedily: while there's at least one uncovered segment, find the one with the smallest r_i and place a point at r_i. Suppose that the largest number of points inside one of the given segments is t in this placement.

It turns out that the answer is either t or $t-1$. Indeed, consider points $x_1, x_2, \ldots, x_t$ inside the segment $[l_i, r_i]$ with the largest number of points. Consider the rightmost point with coordinate less than l_i in the optimal placement. Then the next point to the right of it has to be at coordinate at most x_2, the second next point has to be at coordinate at most x_3, ..., the $t-1$-th next point has to be at coordinate at most x_t. Hence, segment $[l_i, r_i]$ will contain at least $t-1$ points.

It remains to check if the answer is $t-1$. Let's compress the ends of segments and go through points from right to left. For each point x, let's answer the following question: if this point is included into the set, is it possible to include some points with coordinates more than x so that each segment with $r_i \geq x$ contains at least one point, and each segment contains at most $t-1$ points? We'll call points satisfying this condition *legal*.

For each point x, let w_x be the rightmost legal point such that there are no segments strictly inside $[x, w_x]$. To check the condition for a particular point x, let's start with finding w_x. After that, find $y = w(w(\ldots w(x)\ldots))$ ($t-1$ times). If there exists a segment containing both points x and y, this segment would contain t points if point x was placed, thus, point x is illegal. Otherwise, point x is legal.

Finally, if the point to the left of all segments is legal, we can form a sequence of legal points that is a valid solution for $t-1$, otherwise, the answer is t and we output the initial greedy placement.