F — Fortune From Folly

Problem

In infinite random binary sequence $x_1, x_2, x_3, \ldots$ where each $x_i = 1$ with probability p (independently), what is expected first value of i such that $x_i + x_{i-1} + \ldots + x_{i-n+1} \geq k$?

Solution

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Statistics at 4-hour mark: 57 submissions, 27 accepted, first after 00:31