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- Assign a random hash value $h(x)$ to each array value $x$.
- Define hash $h\left(A_{i}\right)$ of a prefix to be $\sum_{j=1}^{i} h\left(a_{j}\right)$.
- If no hash collisions then $A_{i}$ is a permutation of $B_{i}$ if and only if $h\left(A_{i}\right)=h\left(B_{i}\right)$.


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Statistics at 4-hour mark: 11 submissions, 5 accepted, first after 01:51

