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- ③ So this takes $O(a \cdot b)$ time which is fast enough because a and b are very small.

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Statistics at 4-hour mark: 374 submissions, 185 accepted, first after 00:02