Problem F
Faulty Factorial

Submits: 229
Accepted: at least 32

First solved by: UW3
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00:38:14

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Faulty factorial: Take any factor of a factorial and make it smaller, but keep it positive.

Factorial: $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$

Faulty factorial: $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \mathbf{2} \cdot 7 \cdot 8$

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If $n < p$:

None of the factors is divisible by p: impossible.

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Else:

The factorial is already divisible by p, just don’t mess it up. Impossible when $n = p = 2$.

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Case r > 0:

- If n >= 2p:
  - Two factors divisible by p, we can't make both smaller: impossible.

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- Else:
  - n < p <= 10000000, so we can try each factor.

Problem: Find a faulty factorial of length $n < p$, with a fault at position i, that gives reminder $r > 0$ when divided by prime number p.

We are looking for x such that:

$$n! / i \cdot x \equiv r \qquad \text{(modulo } p\text{)}$$

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$$x \equiv r \cdot i \cdot n!^{p-2} \quad \text{(modulo p)}$$

Compute x, and check whether $x < i$.