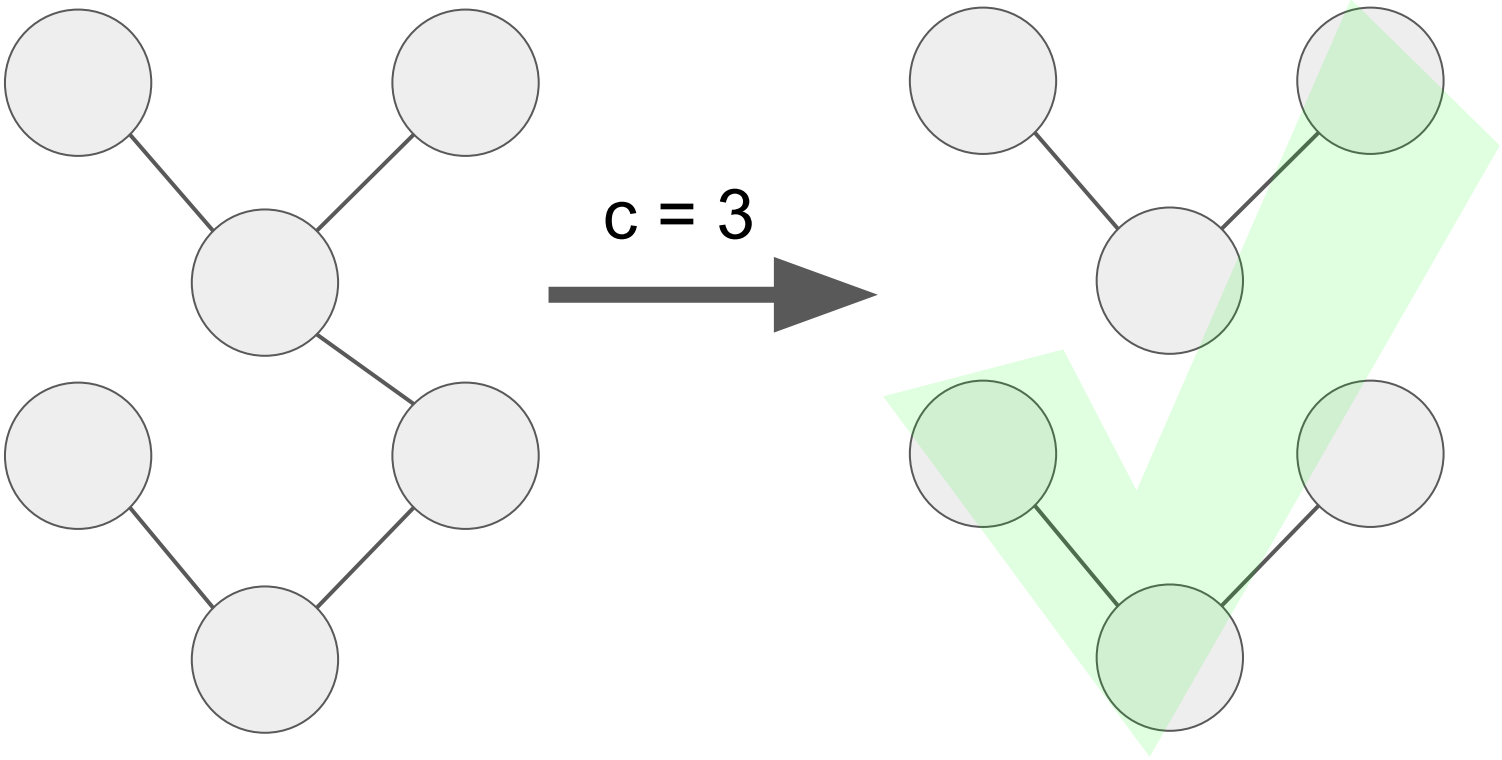
Problem J
Justified Jungle

Submits: 203
Accepted: at least 17

First solved by: Jagiellonian 1
Jagiellonian University in Krakow
(Hlembotskyi, Stokowacki, Zieliński)
00:16:32

Author: Luka Kalinovčić, Ivan Katanić

Problem: Given a tree, find all integers c, such that we can cut a tree into components of size c.



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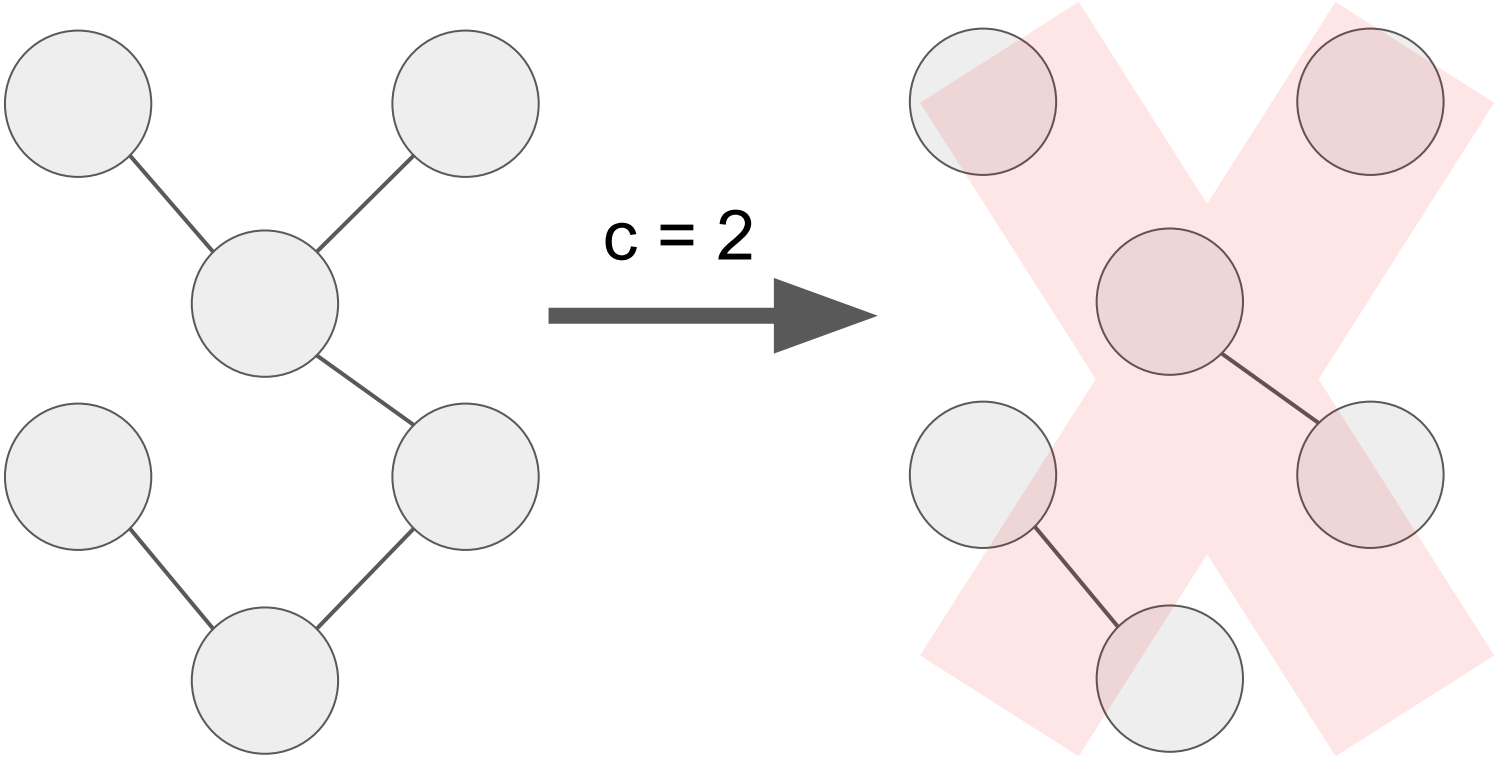

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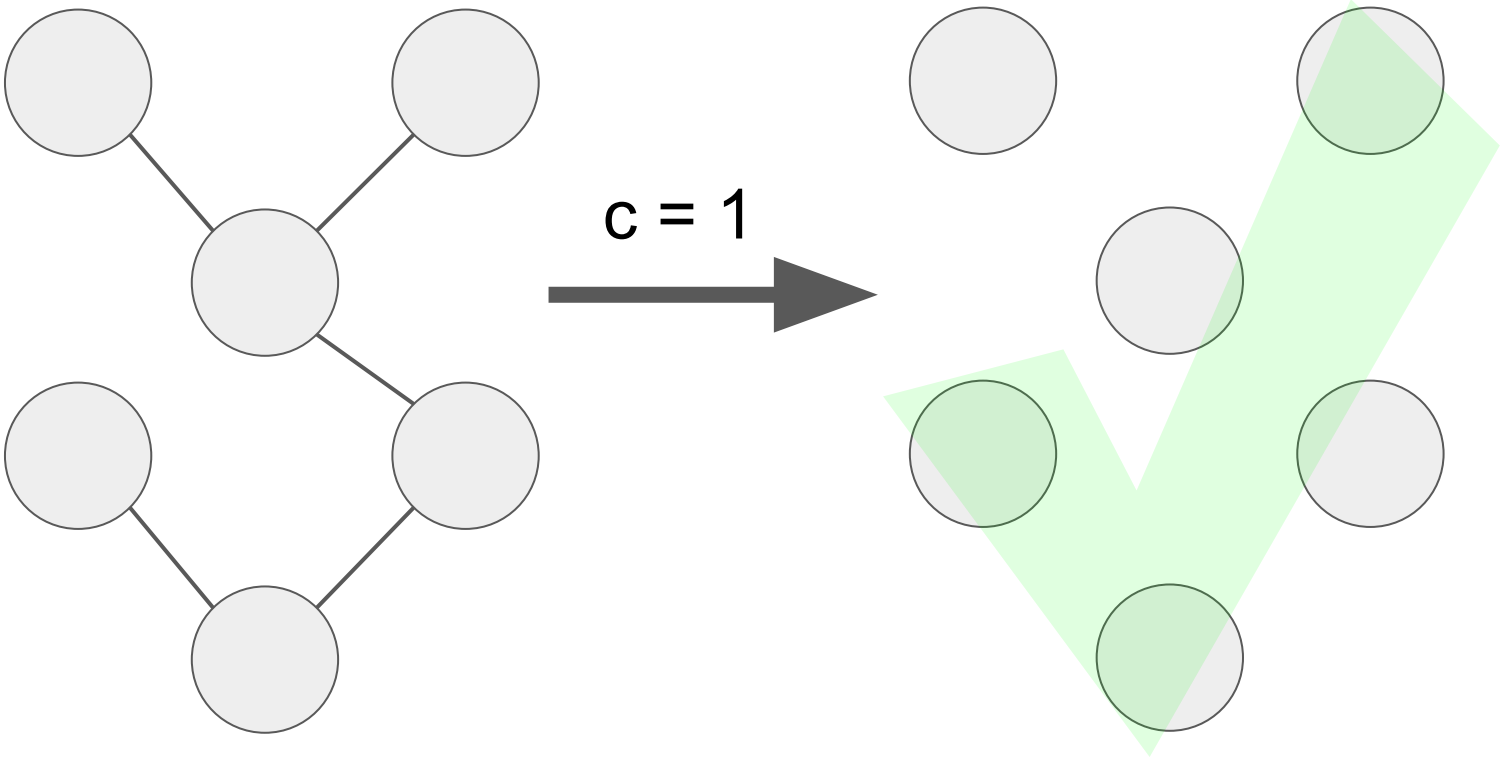

Problem: Given a tree, find all integers c, such that we can cut a tree into components of size c.

The tree size needs to be divisible by c.
There aren't that many divisors: worst case 240 for n=720720.
We can try each divisor separately.

Problem: Given a tree of size n and integer c, such that c | n, can we cut it into components of size c?

Iterative algorithm:
If n = c: done.
Otherwise:

- Find an edge that divides the tree into subtrees of sizes c and n − c.
- If there is no such edge: impossible.
- Otherwise: Cut the edge and repeat the algorithm on the subtree of size n − c.

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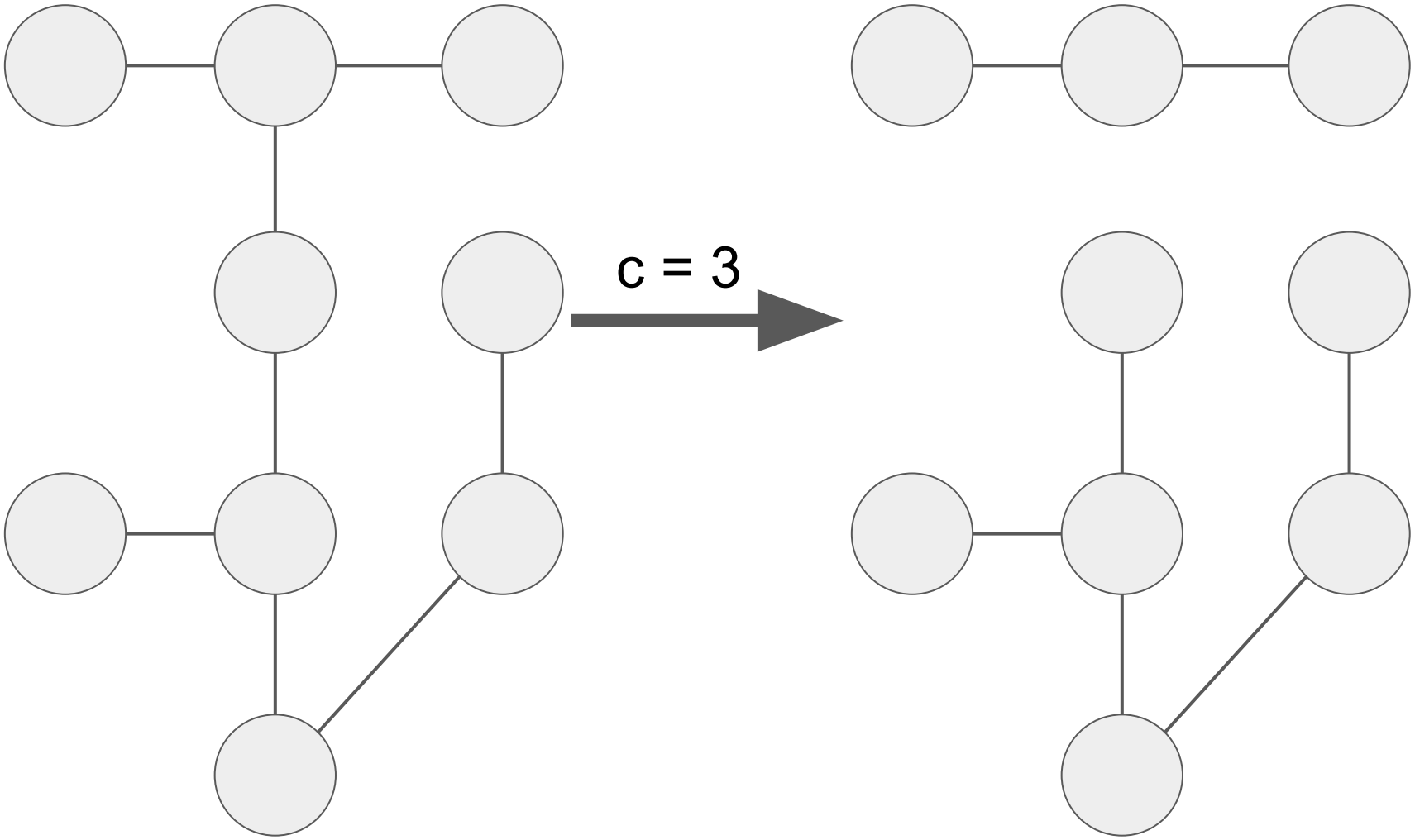

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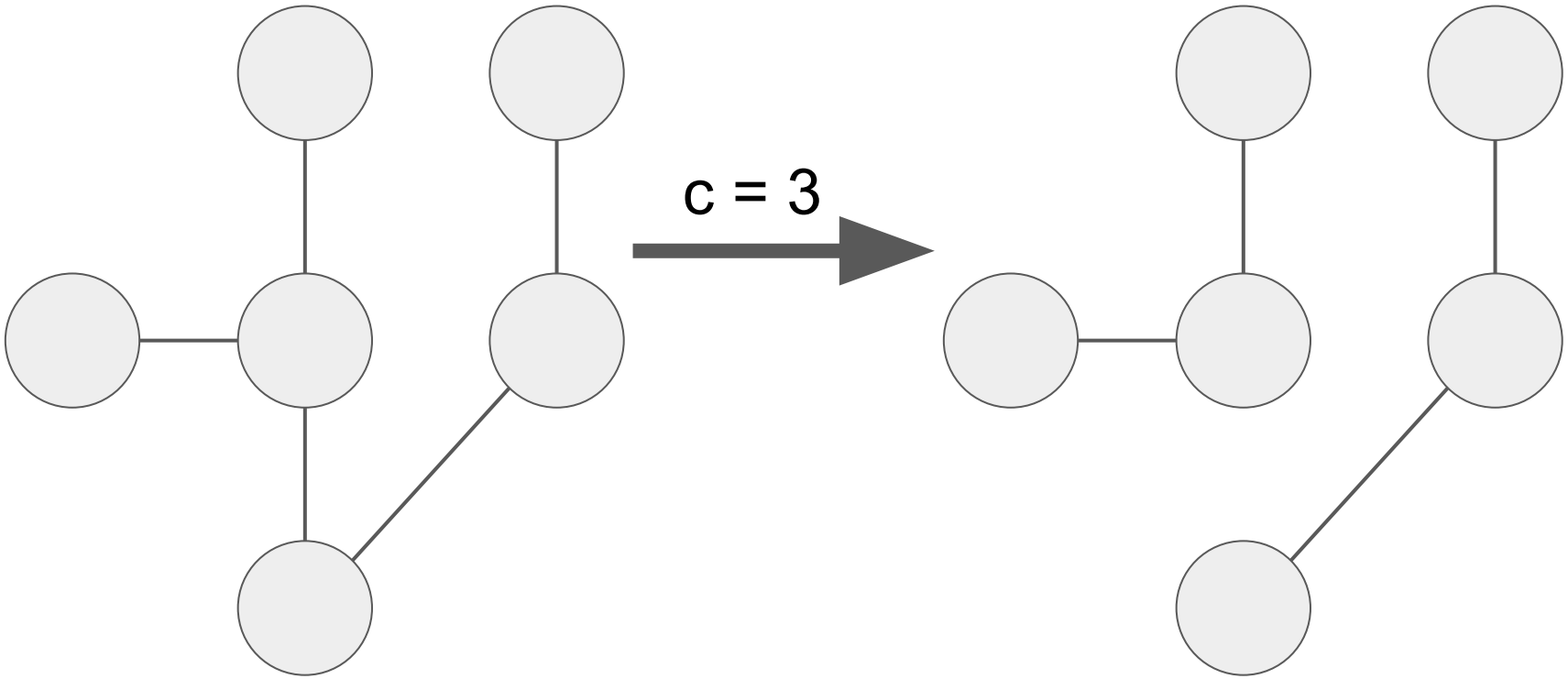

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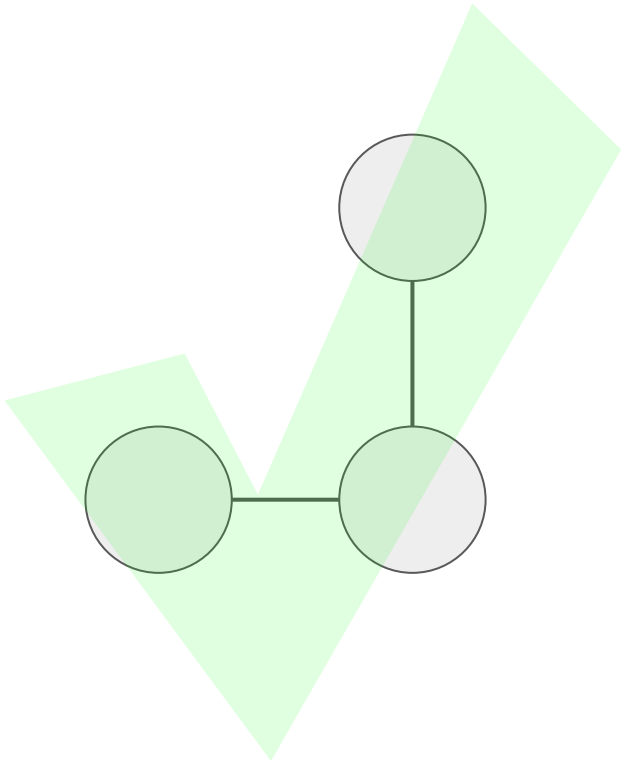

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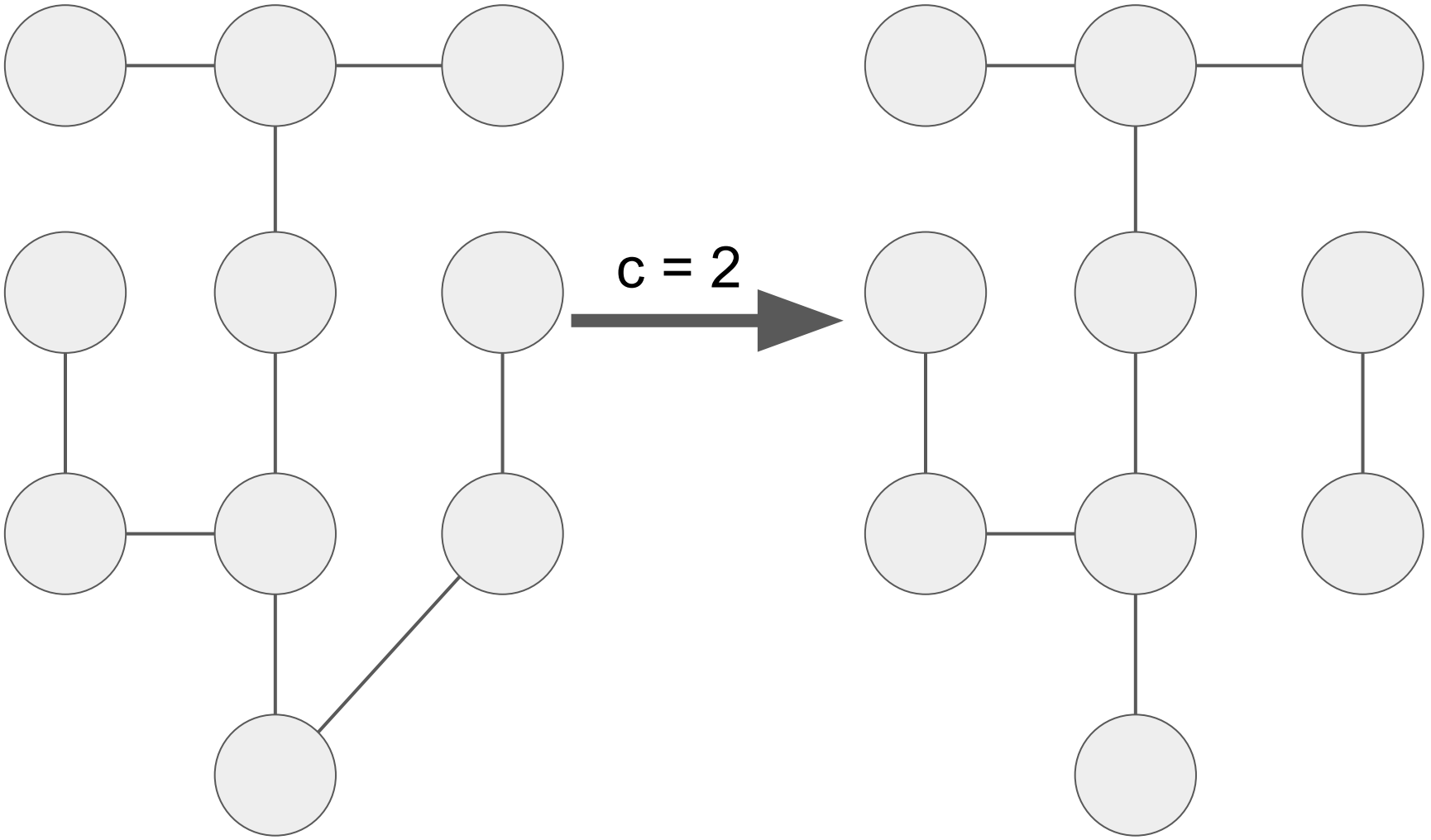

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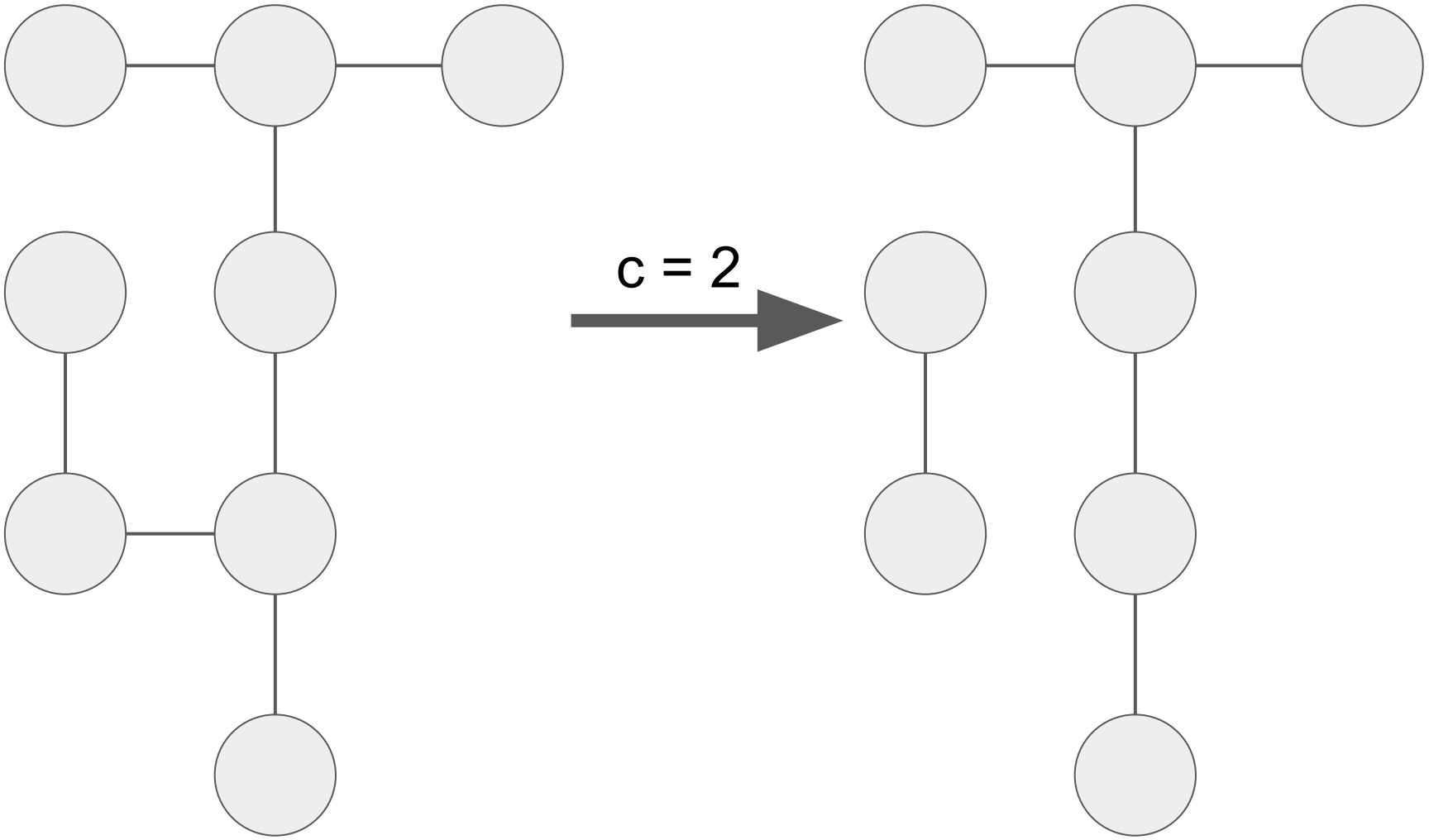

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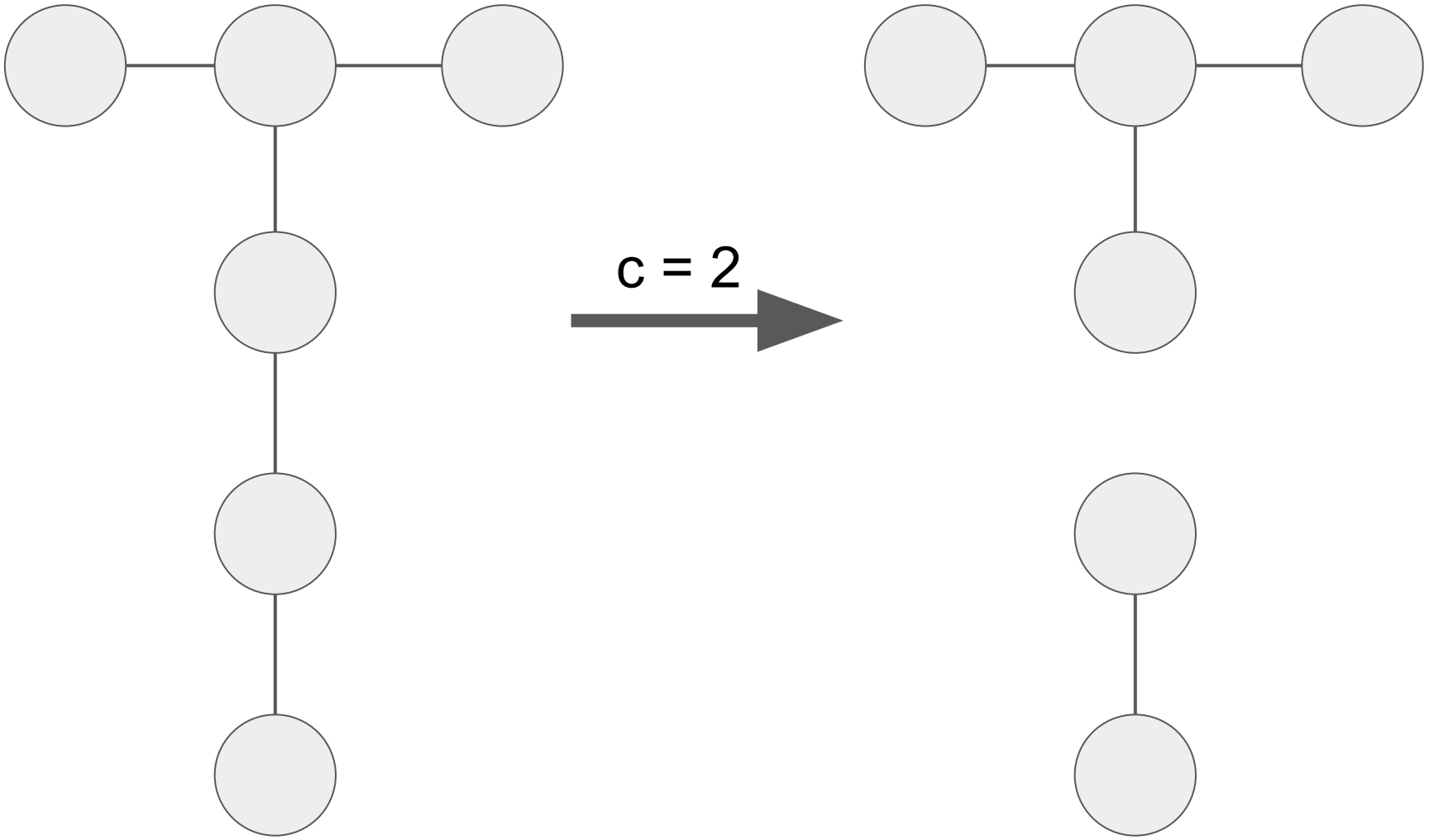

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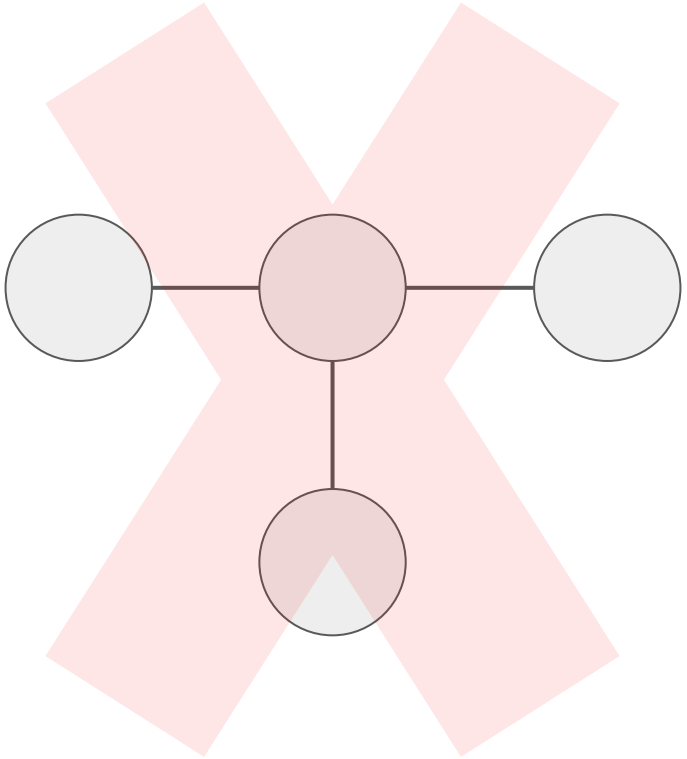


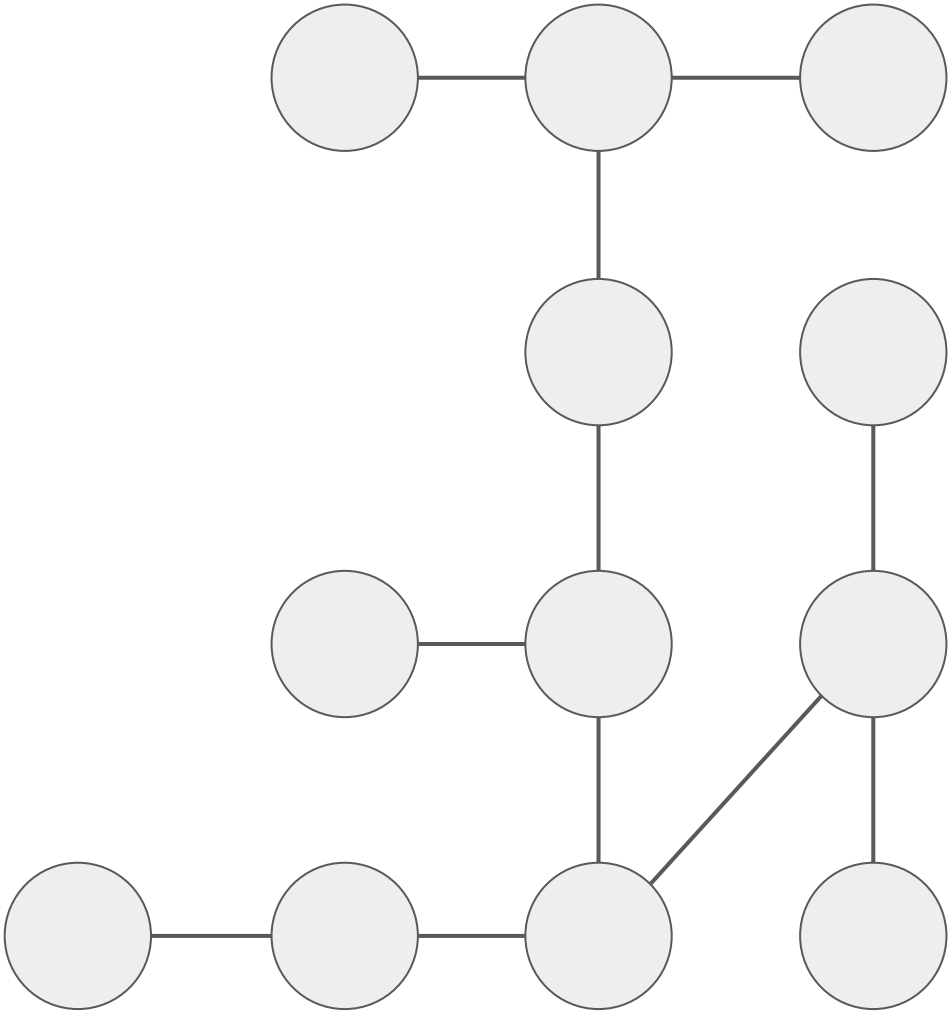
Problem: Given a tree of size n and integer c, such that c | n, can we cut it into components of size c?

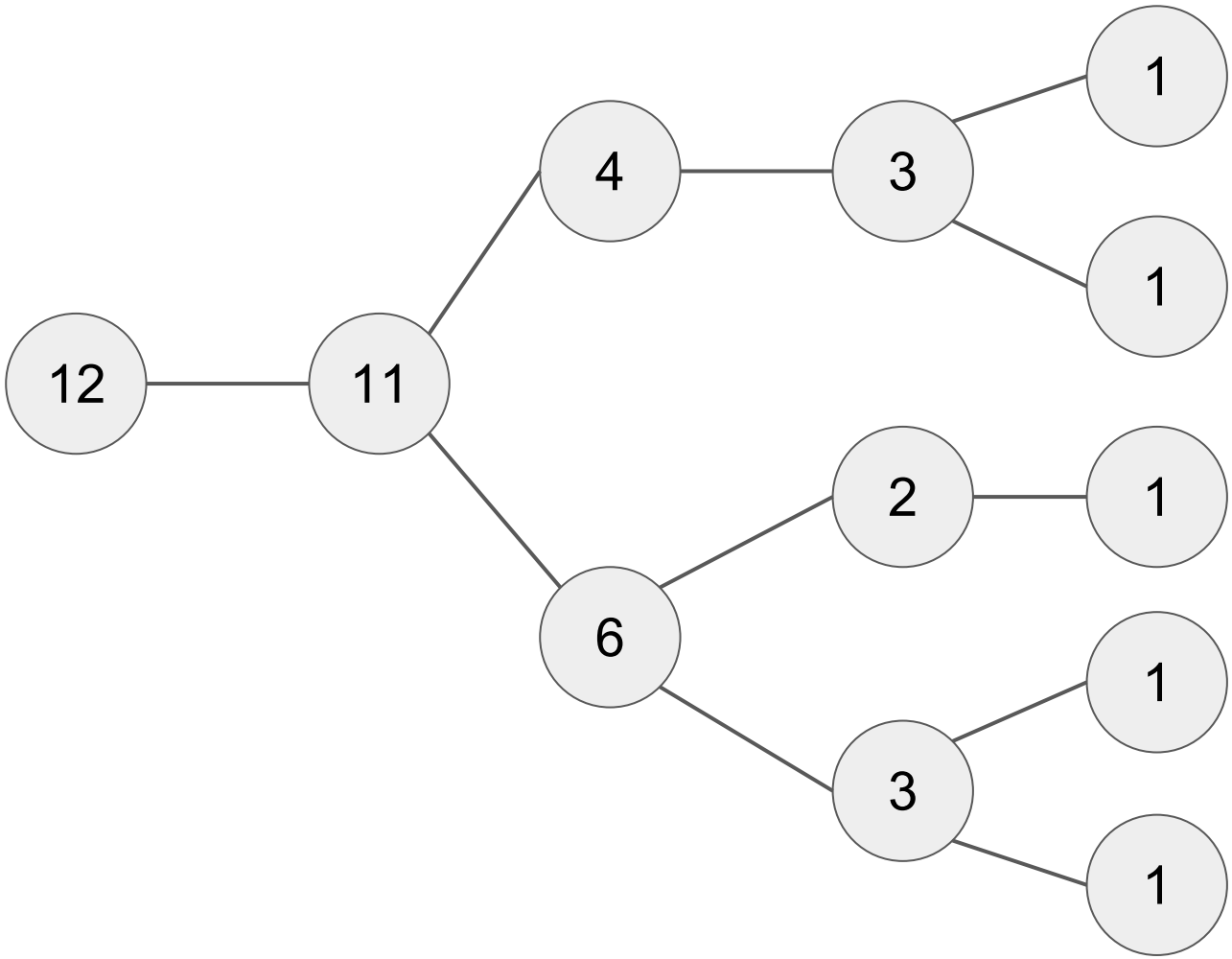
Iterative algorithm is difficult to implement in O(n), and might time out.

Simplified algorithm:
Root the tree and compute the size of each subtree (only once, no need to repeat for each divisor).

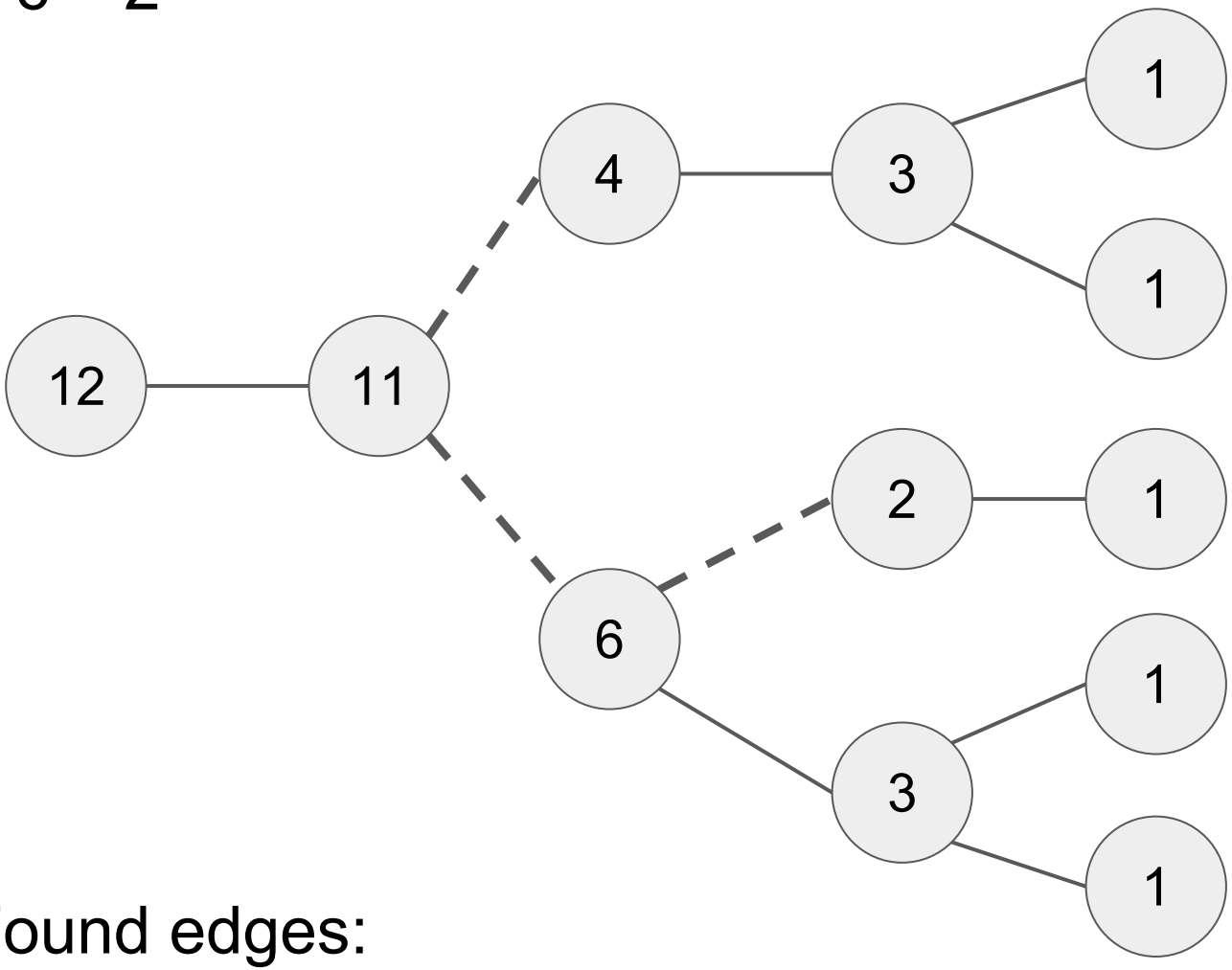
Find edges with subtrees sizes equal to a multiple of c. Those are the ones we'll end up cutting.

If the number of found edges is equal to n / c − 1: yes!
Otherwise: no!




12
11
4
3
1
1
6
2
1
3
1
1

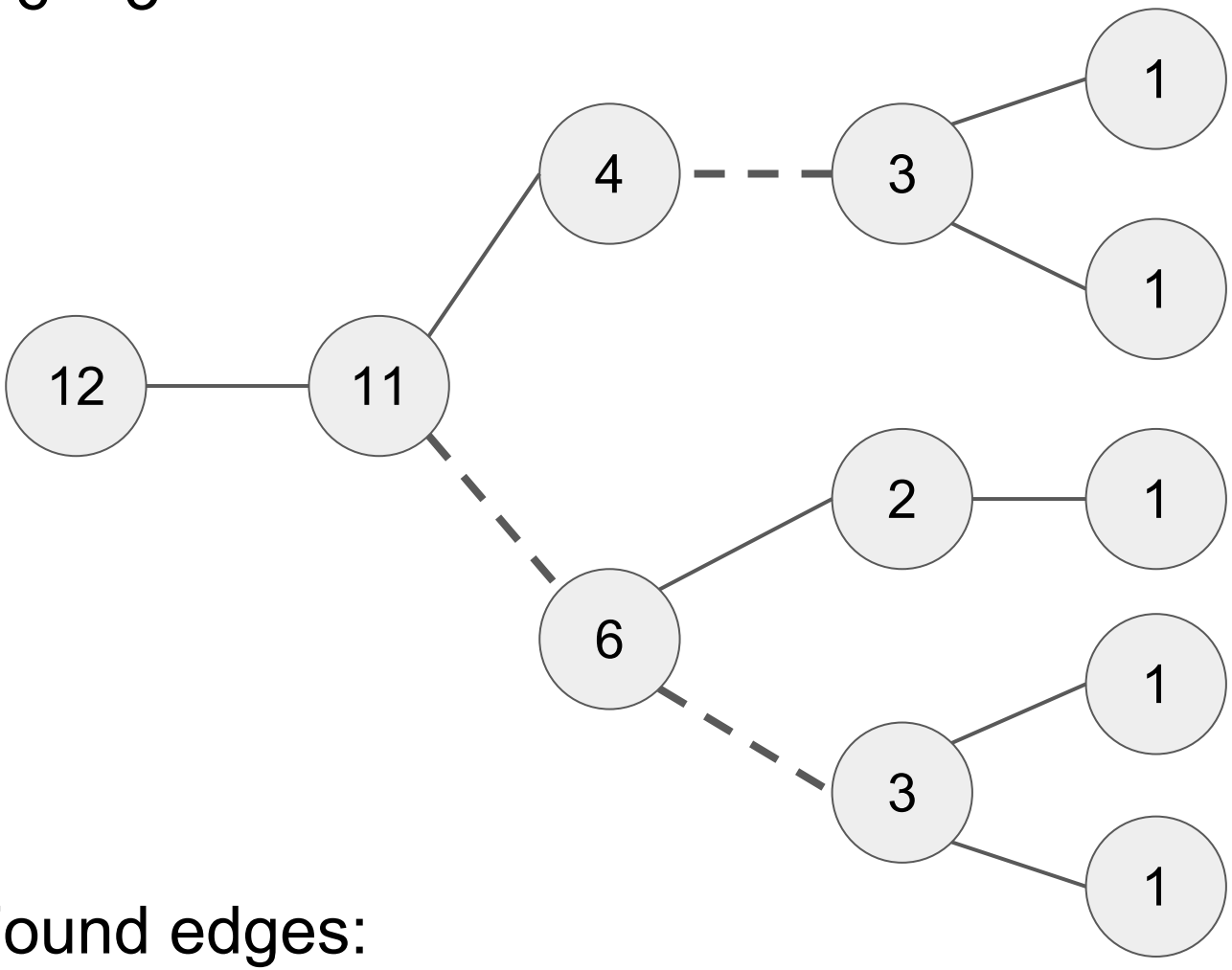
c = 2



Found edges:

$3 \neq n / c - 1 \rightarrow$ NO

c = 3



Found edges:

3 = n / c − 1 → YES

Overall complexity: $O(n \cdot \sigma(n))$, where $\sigma(n)$ is the number of divisors of n.

$O(n + \sigma(n)^2)$ is possible with an extra insight.