

# Keys

Timothy the architect has designed a new escape game. In this game, there are  $n$  rooms numbered from  $0$  to  $n - 1$ . Initially, each room contains exactly one key. Each key has a type, which is an integer between  $0$  and  $n - 1$ , inclusive. The type of the key in room  $i$  ( $0 \leq i \leq n - 1$ ) is  $r[i]$ . Note that multiple rooms may contain keys of the same type, i.e., the values  $r[i]$  are not necessarily distinct.

There are also  $m$  **bidirectional** connectors in the game, numbered from  $0$  to  $m - 1$ . Connector  $j$  ( $0 \leq j \leq m - 1$ ) connects a pair of different rooms  $u[j]$  and  $v[j]$ . A pair of rooms can be connected by multiple connectors.

The game is played by a single player who collects the keys and moves between the rooms by traversing the connectors. We say that the player **traverses** connector  $j$  when they use this connector to move from room  $u[j]$  to room  $v[j]$ , or vice versa. The player can only traverse connector  $j$  if they have collected a key of type  $c[j]$  before.

At any point during the game, the player is in some room  $x$  and can perform two types of actions:

- collect the key in room  $x$ , whose type is  $r[x]$  (unless they have collected it already),
- traverse a connector  $j$ , where either  $u[j] = x$  or  $v[j] = x$ , if the player has collected a key of type  $c[j]$  beforehand. Note that the player **never** discards a key they have collected.

The player **starts** the game in some room  $s$  not carrying any keys. A room  $t$  is **reachable** from a room  $s$ , if the player who starts the game in room  $s$  can perform some sequence of actions described above, and reach room  $t$ .

For each room  $i$  ( $0 \leq i \leq n - 1$ ), denote the number of rooms reachable from room  $i$  as  $p[i]$ . Timothy would like to know the set of indices  $i$  that attain the minimum value of  $p[i]$  across  $0 \leq i \leq n - 1$ .

## Implementation Details

You are to implement the following procedure:

```
int[] find_reachable(int[] r, int[] u, int[] v, int[] c)
```

- $r$ : an array of length  $n$ . For each  $i$  ( $0 \leq i \leq n - 1$ ), the key in room  $i$  is of type  $r[i]$ .
- $u, v$ : two arrays of length  $m$ . For each  $j$  ( $0 \leq j \leq m - 1$ ), connector  $j$  connects rooms  $u[j]$  and  $v[j]$ .
- $c$ : an array of length  $m$ . For each  $j$  ( $0 \leq j \leq m - 1$ ), the type of key needed to traverse connector  $j$  is  $c[j]$ .

- This procedure should return an array  $a$  of length  $n$ . For each  $0 \leq i \leq n - 1$ , the value of  $a[i]$  should be 1 if for every  $j$  such that  $0 \leq j \leq n - 1$ ,  $p[i] \leq p[j]$ . Otherwise, the value of  $a[i]$  should be 0.

## Examples

### Example 1

Consider the following call:

```
find_reachable([0, 1, 1, 2],
               [0, 0, 1, 1, 3], [1, 2, 2, 3, 1], [0, 0, 1, 0, 2])
```

If the player starts the game in room 0, they can perform the following sequence of actions:

Current room	Action
0	Collect key of type 0
0	Traverse connector 0 to room 1
1	Collect key of type 1
1	Traverse connector 2 to room 2
2	Traverse connector 2 to room 1
1	Traverse connector 3 to room 3

Hence room 3 is reachable from room 0. Similarly, we can construct sequences showing that all rooms are reachable from room 0, which implies  $p[0] = 4$ . The table below shows the reachable rooms for all starting rooms:

Starting room $i$	Reachable rooms	$p[i]$
0	[0, 1, 2, 3]	4
1	[1, 2]	2
2	[1, 2]	2
3	[1, 2, 3]	3

The smallest value of  $p[i]$  across all rooms is 2, and this is attained for  $i = 1$  or  $i = 2$ . Therefore, this procedure should return [0, 1, 1, 0].

### Example 2

```
find_reachable([0, 1, 1, 2, 2, 1, 2],
               [0, 0, 1, 1, 2, 3, 3, 4, 4, 5],
               [1, 2, 2, 3, 3, 4, 5, 5, 6, 6],
               [0, 0, 1, 0, 0, 1, 2, 0, 2, 1])
```

The table below shows the reachable rooms:

Starting room $i$	Reachable rooms	$p[i]$
0	[0, 1, 2, 3, 4, 5, 6]	7
1	[1, 2]	2
2	[1, 2]	2
3	[3, 4, 5, 6]	4
4	[4, 6]	2
5	[3, 4, 5, 6]	4
6	[4, 6]	2

The smallest value of  $p[i]$  across all rooms is 2, and this is attained for  $i \in \{1, 2, 4, 6\}$ . Therefore, this procedure should return [0, 1, 1, 0, 1, 0, 1].

### Example 3

```
find_reachable([0, 0, 0], [0], [1], [0])
```

The table below shows the reachable rooms:

Starting room $i$	Reachable rooms	$p[i]$
0	[0, 1]	2
1	[0, 1]	2
2	[2]	1

The smallest value of  $p[i]$  across all rooms is 1, and this is attained when  $i = 2$ . Therefore, this procedure should return [0, 0, 1].

## Constraints

- $2 \leq n \leq 300\,000$
- $1 \leq m \leq 300\,000$
- $0 \leq r[i] \leq n - 1$  for all  $0 \leq i \leq n - 1$
- $0 \leq u[j], v[j] \leq n - 1$  and  $u[j] \neq v[j]$  for all  $0 \leq j \leq m - 1$

- $0 \leq c[j] \leq n - 1$  for all  $0 \leq j \leq m - 1$

## Subtasks

1. (9 points)  $c[j] = 0$  for all  $0 \leq j \leq m - 1$  and  $n, m \leq 200$
2. (11 points)  $n, m \leq 200$
3. (17 points)  $n, m \leq 2000$
4. (30 points)  $c[j] \leq 29$  (for all  $0 \leq j \leq m - 1$ ) and  $r[i] \leq 29$  (for all  $0 \leq i \leq n - 1$ )
5. (33 points) No additional constraints.

## Sample Grader

The sample grader reads the input in the following format:

- line 1:  $n \ m$
- line 2:  $r[0] \ r[1] \ \dots \ r[n - 1]$
- line  $3 + j$  ( $0 \leq j \leq m - 1$ ):  $u[j] \ v[j] \ c[j]$

The sample grader prints the return value of `find_reachable` in the following format:

- line 1:  $a[0] \ a[1] \ \dots \ a[n - 1]$