

**2020/2021 SOUTHERN CALIFORNIA REGIONAL  
INTERNATIONAL COLLEGIATE PROGRAMMING CONTEST**

**Problem 9  
Redundant Binary Notation**

Redundant binary notation is similar to binary notation, except instead of allowing only 0s and 1s for each digit, we allow any integer digit in the range  $[0, t]$ , where  $t$  is some specified upper bound. For example, if  $t = 2$ , the digit 2 is permitted, and we may write the decimal number 4 as 100, 20, or 12. If  $t = 1$ , every number has precisely one representation, which is its typical binary representation. In general, if a number is written as  $d_l d_{l-1} \dots d_1 d_0$  in redundant binary notation, the equivalent decimal number is  $d_l \cdot 2^l + d_{l-1} \cdot 2^{l-1} + \dots + d_1 \cdot 2^1 + d_0 \cdot 2^0$ .

Redundant binary notation can allow carryless arithmetic, and thus has applications in hardware design and even in the design of worst-case data structures. For example, consider insertion into a standard binomial heap. This operation takes  $O(\log n)$  worst-case time but  $O(1)$  amortized time. This is because the binary number representing the total number of elements in the heap can be incremented in  $O(\log n)$  worst-case time and  $O(1)$  amortized time. By using a redundant binary representation of the individual binomial trees in a binomial heap, it is possible to improve the worst-case insertion time of binomial heaps to  $O(1)$ .

However, none of that information is relevant to this question. In this question, your task is simple. Given a decimal number  $N$  and the digit upper bound  $t$ , you are to count the number of possible representations  $N$  has in redundant binary notation with each digit in range  $[0, t]$  with no leading zeros.

Input consists of a single line with two decimal integers  $N$  ( $0 \leq N \leq 10^{16}$ ) and  $t$  ( $1 \leq t \leq 100$ ), separated by whitespace.

Output in decimal the number of representations the decimal number  $N$  has in redundant binary notation with each digit in range  $[0, t]$  with no leading zeros. Since the number of representations may be very large, output the answer modulo the large prime 998 244 353.

*Sample Input 1*

4 2

*Output for Sample Input 1*

3

*Sample Input 2*

6 3

*Output for Sample Input 2*

4

**Problem 9**  
**Redundant Binary Notation (continued)**

*Sample Input 3*

479 1

*Output for Sample Input 3*

1

*Sample Input 4*

3846927384799 62

*Output for Sample Input 4*

690163857

*Sample Input 5*

549755813887 2

*Output for Sample Input 5*

1