

## Problem I

### Ranks

**Time Limit: 3 seconds**

A *finite field*  $\mathbf{F}_2$  consists of two elements: 0 and 1. Addition and multiplication on  $\mathbf{F}_2$  are those on integers modulo two, as defined below.

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

A set of vectors  $\mathbf{v}_1, \dots, \mathbf{v}_k$  over  $\mathbf{F}_2$  with the same dimension is said to be *linearly independent* when, for  $c_1, \dots, c_k \in \mathbf{F}_2$ ,  $c_1\mathbf{v}_1 + \dots + c_k\mathbf{v}_k = \mathbf{0}$  is equivalent to  $c_1 = \dots = c_k = 0$ , where  $\mathbf{0}$  is the zero vector, the vector with all its elements being zero.

The *rank* of a matrix is the maximum cardinality of its linearly independent sets of column vectors. For example, the rank of the matrix  $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$  is two; the column vectors  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  (the first and the third columns) are linearly independent while the set of all three column vectors is *not* linearly independent. Note that the rank is zero for the zero matrix.

Given the above definition of the rank of matrices, the following may be an intriguing question. *How does a modification of an entry in a matrix change the rank of the matrix?* To investigate this question, let us suppose that we are given a matrix  $A$  over  $\mathbf{F}_2$ . For any indices  $i$  and  $j$ , let  $A^{(ij)}$  be a matrix equivalent to  $A$  except that the  $(i, j)$  entry is flipped.

$$A_{kl}^{(ij)} = \begin{cases} A_{kl} + 1 & (k = i \text{ and } l = j) \\ A_{kl} & (\text{otherwise}) \end{cases}$$

In this problem, we are interested in the rank of the matrix  $A^{(ij)}$ . Let us denote the rank of  $A$  by  $r$ , and that of  $A^{(ij)}$  by  $r^{(ij)}$ . Your task is to determine, for all  $(i, j)$  entries, the relation of ranks before and after flipping the entry out of the following possibilities: (i)  $r^{(ij)} < r$ , (ii)  $r^{(ij)} = r$ , or (iii)  $r^{(ij)} > r$ .

### Input

The input consists of a single test case of the following format.

```

n m
A11 ... A1m
⋮
An1 ... Anm

```

$n$  and  $m$  are the numbers of rows and columns in the matrix  $A$ , respectively ( $1 \leq n \leq 1000$ ,  $1 \leq m \leq 1000$ ). In the next  $n$  lines, the entries of  $A$  are listed without spaces in between.  $A_{ij}$  is the entry in the  $i$ -th row and  $j$ -th column, which is either 0 or 1.

## Output

Output  $n$  lines, each consisting of  $m$  characters. The character in the  $i$ -th line at the  $j$ -th position must be either - (minus), 0 (zero), or + (plus). They correspond to the possibilities (i), (ii), and (iii) in the problem statement respectively.

**Sample Input 1**

```
2 3
001
101
```

**Sample Output 1**

```
-0-
-00
```

**Sample Input 2**

```
5 4
1111
1000
1000
1000
1000
```

**Sample Output 2**

```
0000
0+++
0+++
0+++
0+++
```

**Sample Input 3**

```
10 10
1000001001
0000010100
0000100010
0001000001
0010000010
0100000100
1000001000
0000010000
0000100000
0001000001
```

**Sample Output 3**

```
000-00000-
0-00000-00
00-00000-0
+00000+000
00-0000000
0-00000000
000-00000-
0-000-0-00
00-0-000-0
+00000+000
```

**Sample Input 4**

```
1 1
0
```

**Sample Output 4**

```
+
```