Problem D
Distance and Tree

Time limit: 3 seconds

Memory limit: 1024 megabytes

Problem Description

Graph problems are popular in competitive programming, and problems related to distanceis and trees appear frequently. Let us start with some definitions.

A *set* is a collection of distinct elements. An *undirected simple graph* G is a pair (V, E), where V is a set and E is a set of unordered pairs of V's elements. For a graph $G = (V, E)$, we call V as G's vertex set and E as G's edge set. Elements in V are vertices, and elements in E are edges.

Let u and v be vertices in V. A *path* from u to v of length k is a sequence of edges $e_1, e_2, \ldots, e_k \in E$ such that there exists a sequence of distinct vertices, $v_1, \ldots, v_{k+1}$, satisfying the following conditions.

- $u = v_1$.
- $v = v_{k+1}$.
- $e_i = \{v_i, v_{i+1}\}$.

If p is a path from u to v, then u and v are *connected* by p.

We can define distances and trees now. Given two vertices $u, v \in V$, the *distance* $\delta(u, v)$ from u to v is 0 if $u = v$. If there exists a path from u to v, then $\delta(u, v)$ is the minimum number of edges required to form a path from u to v. Otherwise, $\delta(u, v) = \infty$. A *tree* is an undirected graph in which any distinct two vertices u and v are connected by exactly one path.

Danny gives you a sequence of non-negative integers $d_1, d_2, \ldots, d_n$ and asks you to construct a tree $G_T = (V_T, E_T)$ satisfying the following conditions.

- The vertex set $V_T = \{p_1, \ldots, p_n\}$ is a set of points on a two dimensional Euclidean plane. For $1 \le k \le n$, the coordinate of p_k is $(\cos k\theta, \sin k\theta)$ where $\theta = \frac{2\pi}{n}$.
- For any two distinct edges $\{p_a, p_b\}$ and $\{q_a, q_b\}$ in E_T, the line segments $\overline{p_a p_b}$ and $\overline{q_a q_b}$ do not intersect unless those two edges share a common vertex (that is, $\{p_a, p_b\} \cap \{q_a, q_b\} \neq \emptyset$).
- There exists a vertex r such that $\delta(r, p_k) = d_k$ for $1 \le k \le n$. We call r as the root of G_T.

If there exists such tree graph, please output the edge set E_T. Otherwise, output `-1`.

Input Format

The first line contains a positive integer n indicating the number of vertices of the tree to be constructed. The second line contains n non-negative integers $d_1, \ldots, d_n$, the sequence given by Danny.

Output Format

If there does not exist such a tree G_T, output `-1`. Otherwise, output $n-1$ lines to represent the edge set E_T. The i-th line should contain two space-separated integers u_i and v_i. The i-th edge in E_T should be $\{p_{u_i}, p_{v_i}\}$. If there are multiple solutions, you may output any of them.

Technical Specification

- $2 \leq n \leq 100000$
- For $1 \leq k \leq n$, $0 \leq d_k \leq n-1$.

Sample Input 1

```
5
0 1 2 1 3
```

Sample Output 1

```
-1
```

Sample Input 2

```
5
1 1 0 1 1
```

Sample Output 2

```
1 3
3 2
3 4
5 3
```