Uni Cup

## Problem B. Equation Discovering

Input file:	standard input
Output file:	standard output
Time limit:	2 seconds
Memory limit:	1024 megabytes

Professor Mika is a researcher in computer science who posed the following question: given n pairs of (x, y) where  $1 \le n \le 20$ , how can we find the governing equation y = f(x) that fits all the pairs? In other words, he seeks to determine an equation involving binary operators  $(+, -, \times, \div)$ , unary operators (sin, cos), symbol x, and parentheses that fits all the given pairs, e.g.,  $y = x \times x \div \sin(x)$  or  $y = x \times (x + x \div x)$ .

To generate all the valid equations, we define a context-free grammar as follows:

- 1. The start symbol is S.
- 2.  $S \rightarrow S + S \mid S S$
- 3.  $S \to S \times S \mid S \div S$
- 4.  $S \to \sin(S) \mid \cos(S)$
- 5.  $S \to (S) \mid x$

However, to prevent overfitting, we limit the complexity of the equations to be less than or equal to 9, where complexity is defined as two times the number of binary operators  $(+, -, \times, \div)$  plus one times the number of unary operators (sin, cos) in the equation. For example, the equation  $x + (x + x \times x)$  has a complexity of 6, while  $x \times \sin(x)$  has a complexity of 3. Only equations with a complexity less than or equal to 9 will be considered correct.

## Input

The first line contains one integer n  $(1 \le n \le 20)$ , denoting the number of (x, y) pairs to be fit.

The following n lines, each line two real numbers x, y ( $|x|, |y| < 10^3$ ) with exactly six digits after the decimal point, means the (x, y) pairs.

The value of x is guaranteed to be accurate, and we use some valid equation to generate the value of y, then round it to six digits.

## Output

The output has only one line of expression f, consists of '+', '-', '\*' (for  $\times$ ), '/' (for  $\div$ ), 'sin', 'cos', 'x', '(' and ')'.

Your answer will be considered correct if it satisfies the following conditions:

- 1. The expression f is generated using the context-free grammar described earlier and is valid according to that grammar.
- 2. The expression f has a complexity of no more than 9 and does not exceed 1000 characters in length.
- 3. For each (x, y) pair, the absolute or relative error between f(x) and y is no greater than  $10^{-3}$ . i.e.  $\frac{|f(x)-y|}{\max(1,|y|)} \leq 10^{-3}$ .
- 4. When calculating the division operation, the absolute value of the dividend must be no less than 0.01.



The evaluation of f(x) follows the standard mathematical conventions, where expressions within parentheses are evaluated first, followed by multiplication and division (with higher priority than addition and subtraction), and all binary operators are left-associative.

The equation we used to generate the data is guaranteed to satisfy all these four requirements in tests. What's more, to avoid possible floating point problems, for requirement four, we promise the solution meets a stricter bound of 0.02.

Note that there may be multiple valid solutions to this problem. Each of them would be accepted.

## Examples

standard input	standard output
3	x*x
1.000000 1.000000	
2.000000 4.000000	
3.000000 9.000000	
3	sin(x)/(x*x)
0.618000 1.517072	
0.314000 3.132637	
1.414000 0.494016	