

2023 Canadian Computing Olympiad

Day 2, Problem 3

Triangle Collection

Time Limit: 4 seconds

Problem Description

Alice has a collection of sticks. Initially, she has c_ℓ sticks of length ℓ for each $\ell = 1, \dots, N$.

Alice would like to use her sticks to make some isosceles triangles. An isosceles triangle is made of two sticks of the same length, say ℓ , and a third stick with a length between 1 and $2\ell - 1$ inclusive. Note that the triangles must strictly obey the triangle inequality, and equilateral triangles are okay. Each stick may be used in at most one triangle. Alice would like to know the maximum number of isosceles triangles she can make with her sticks.

There are Q events that change the collection of sticks she has. The i -th event consists of two integers ℓ_i and d_i , representing that the number of sticks of length ℓ_i changes by d_i . Note that d_i may be positive, negative, or even 0, but Alice will never have a negative number or more than 10^9 sticks of each length.

Your task is to determine the maximum number of isosceles triangles Alice can make after each event if she uses her sticks optimally.

Input Specification

The first line of input contains two space-separated integers N and Q .

The second line of input contains N space-separated integers c_1, c_2, \dots, c_N ($0 \leq c_i \leq 10^9$), representing Alice's initial collection.

The next Q lines of input each contain two space-separated integers ℓ_i and d_i ($1 \leq \ell_i \leq N$, $-10^9 \leq d_i \leq 10^9$), representing an event.

Initially and after each event, the number of sticks of length ℓ is between 0 and 10^9 for all $\ell = 1, \dots, N$.

Marks Awarded	Bounds on N, Q	Additional Constraints
5 marks	$1 \leq N, Q \leq 2\,000$	There are at most 2 000 sticks in total initially and after each event.
5 marks	$1 \leq N, Q \leq 2\,000$	No additional constraints.
5 marks	$1 \leq N, Q \leq 200\,000$	The number of sticks of each length is either 0, 1, or 2 initially and after each event.
5 marks	$1 \leq N, Q \leq 200\,000$	For each event, $ d_i = 1$.
5 marks	$1 \leq N, Q \leq 200\,000$	No additional constraints.

Output Specification

Output Q lines each containing a single integer, the answer after each event.

Sample Input

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4 3
3 1 4 1
3 -3
1 6
2 1
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Output for Sample Input

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1
3
4
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Explanation of Output for Sample Input

After the first event, Alice can make a single triangle with sticks of lengths $(1, 1, 1)$.

After the second event, Alice can make 3 triangles with sticks of lengths $(1, 1, 1)$.

After the third event, Alice can make 3 triangles with sticks of lengths $(1, 1, 1)$ and a triangle with sticks of lengths $(2, 2, 3)$.